The University of Texas at Austin
In-Class Work 4

M378K Introduction to Mathematical Statistics

September 09, 2026

Expectation and variance.

Problem 4.1. *Source: Sample P exam, Problem #481.*
The number of days required for a damage control team to locate and repair a leak in the hull of a ship is modeled by a discrete random variable N. N is uniformly distributed on $\{1, 2, 3, 4, 5\}$.
The cost of locating and repairing a leak is $N^2 + N + 1$.
Calculate the expected cost of locating and repairing a leak in the hull of the ship.

Solution. We need to calculate

$$\mathbb{E}[N^2 + N + 1] = \mathbb{E}[N^2] + \mathbb{E}[N] + 1.$$

By definition

$$\mathbb{E}[N] = 1 \cdot \frac{1}{5} + ... + 5 \cdot \frac{1}{5} = \frac{1}{5} \cdot \frac{5 \cdot 6}{2} = 3,$$
$$\mathbb{E}[N^2] = 1^2 \cdot \frac{1}{5} + ... + 5^2 \cdot \frac{1}{5} = \frac{1}{5} \cdot \frac{5 \cdot 6 \cdot 11}{6} = 11.$$

Our final answer is $3 + 11 + 1 = 15$.

Problem 4.2. *Source: Sample P exam, Problem #458.*
An investor wants to purchase a total of ten units of two assets, A and B, with annual payoffs f per unit purchased of X and Y, respectively. Each asset has the same purchase price per unit. The payoffs are independent random variables with equal expected values and with $\text{Var}\ (X) = 30$ and $\text{Var}\ (Y) = 20$.
Calculate the number of units of asset A the investor should purchase to minimize the variance of the total payoff.

Solution. Let the number of units of asset A be denoted by n. Then, the total payoff will be equal to

$$nX + (10 - n)Y.$$

The variance of the total payoff will be

$$\text{Var}\ [nX + (10 - n)Y].$$

Due to independence of X and Y, using the properties of variance, we get that the above equals

$$n^2\ \text{Var}\ [X] + (10 - n)^2\ \text{Var}\ [Y] = 30n^2 + 20(10 - n)^2 = 50n^2 - 400n + 2000 = 50(n^2 - 8n + 40).$$

We need to minimize the above with respect to n. This is easily done by noticing that $n^2 - 8n + 40$ is a parabola facing up. So, all we need to do is to figure out the position of its "bottom". In the notation you might recall from quadratic equations, it is

$$n^* = -\frac{B}{2A} = -\frac{-8}{2 \cdot 1} = 4.$$

Note: Yes, you could have also differentiated to find the minimum.