The χ^2-Distribution.

- If $Y \sim N(0,1)$, then $\boxed{W = Y^2 \sim \chi^2(df=1)}$

  Its density is $\boxed{f_W(w) = \frac{1}{\sqrt{2\pi w}} e^{-\frac{w}{2}} \mathbb{1}_{(0,\infty)}(w)}$

  Its mgf is $\boxed{m_W(t) = (1-2t)^{-1/2} = \frac{1}{\sqrt{1-2t}}}$

  $1-2t > 0$
  $1 > 2t$
  $\boxed{\frac{1}{2} > t}$

- $Y_1 \sim N(0,1)$, $Y_2 \sim N(0,1)$ independent

  Set $\boxed{W = Y_1^2 + Y_2^2} \sim \chi^2(df=2)$

  Let's find its mgf:

$$m_W(t) = m_{Y_1^2}(t) \cdot m_{Y_2^2}(t)$$
$$= \frac{1}{\sqrt{1-2t}} \cdot \frac{1}{\sqrt{1-2t}} = \boxed{\frac{1}{1-2t}}$$

Example. $Y \sim E(\tau)$

$$m_Y(t) = \mathbb{E}[e^{t \cdot Y}]$$
$$= \int_0^\infty e^{t \cdot y} \cdot \frac{1}{\tau} \cdot e^{-\frac{y}{\tau}} dy =$$
$$= \frac{1}{\tau} \int_0^\infty e^{(t-\frac{1}{\tau})y} dy$$

What if $t \geq \frac{1}{\tau}$? DNE

Only consider $|t| < \frac{1}{\tau}$

$$= \frac{1}{\tau} \cdot \frac{1}{t-\frac{1}{\tau}} e^{(t-\frac{1}{\tau})y} \Big|_{y=0}^{\infty}$$

($t - \frac{1}{\tau} < 0$)

$$= \frac{1}{\tau} \cdot \frac{\tau}{t\cdot\tau - 1} \cdot (0 - 1) = \frac{1}{1-\tau \cdot t}$$

We know that $W \sim \chi^2(df=2) \sim E(\tau=2)$

Def'n. The χ^2-dist'n w/ n degrees of freedom is the dist'n of the sum

$$W = Y_1^2 + Y_2^2 + \cdots + Y_n^2$$

where $Y_i \sim N(0,1)$ for all $i = 1..n$ and independent.

We write $\boxed{W \sim \chi^2(df = n) = \chi^2(n)}$

Note:

$$m_W(w) = \left(\frac{1}{\sqrt{1-2t}}\right)^n = (1-2t)^{-n/2} = \left(\frac{1}{1-2t}\right)^{n/2}$$

The Gamma Dist'n.

Def'n. A random variable Y is said to have the gamma distribution w/ parameters $k > 0$ and $\tau > 0$ if its mgf is of the form

$$m_Y(t) = \left(\frac{1}{1-\tau t}\right)^k$$

(k: shape parameter; τ: scale parameter)

We write: $\boxed{Y \sim \Gamma(k, \tau)}$

Note:

$$\boxed{\begin{aligned} \mathbb{E}[Y] &= k \cdot \tau \\ \mathrm{Var}[Y] &= k \cdot \tau^2 \end{aligned}}$$

Q: Say that $Y \sim \Gamma(1, \tau)$. What is another name for it?

$Y \sim E(\tau)$

Q: Say that $Y \sim \Gamma(\frac{n}{2}, 2)$. What is another name for it?

$Y \sim \chi^2(n)$

Q: $Y_1 \sim \Gamma(k_1, \tau)$, $Y_2 \sim \Gamma(k_2, \tau)$ independent

$$\boxed{Y_1 + Y_2 \sim \Gamma(k = k_1 + k_2, \tau)}$$

The Statistical Set-Up.

Population. e.g., all the people in this class

Sample. e.g., a committee of 4 students

We assume that it's a **REPRESENTATIVE SAMPLE.**

Use the same word for the results of measurement (or polling) from the population w/ an unknown but COMMON dist'n.