The University of Texas at Austin
Homework assignment 9

Introduction to Mathematical Statistics

March 26, 2026

Instructions: Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Bias. MSE.

Problem 9.1. (5 points) Which of the following estimators is **not** unbiased for μ if $Y_1, ..., Y_n$ is a random sample from the normal distribution $N(\mu, \sigma)$:

a. Y_n
b. $\frac{1}{2}(Y_1 + Y_2)$
c. $Y_1 - Y_2 + Y_3$
d. $\bar{Y}$
e. All of the above are unbiased.

Problem 9.2. (5 points) Let $Y_1, ..., Y_n$ be a random sample of size $n \geq 2$, from $N(\mu, \sigma)$ and let the estimators $\hat{\mu}_1$, $\hat{\mu}_2$ and $\hat{\mu}_3$, for μ, be given by

$$\hat{\mu}_1 = Y_1, \hat{\mu}_2 = \frac{1}{2}(Y_1 + Y_2) \text{ and } \hat{\mu}_3 = \bar{Y}.$$

Then, no matter what μ and σ are, we always have

a. MSE $(\hat{\mu}_1) \leq$ MSE $(\hat{\mu}_2) \leq$ MSE $(\hat{\mu}_3)$
b. MSE $(\hat{\mu}_3) \leq$ MSE $(\hat{\mu}_2) \leq$ MSE $(\hat{\mu}_1)$
c. MSE $(\hat{\mu}_3) \leq$ MSE $(\hat{\mu}_1) \leq$ MSE $(\hat{\mu}_2)$
d. MSE $(\hat{\mu}_1) \leq$ MSE $(\hat{\mu}_3) \leq$ MSE $(\hat{\mu}_2)$
e. None of the above.

Problem 9.3. (40 points) Let (Y_1, Y_2) be a random sample (of size $n = 2$) from the uniform distribution $U(0, \theta)$, with $\theta > 0$ unknown.

i. $(2 + 3 + 10 = 15$ points) Find constants c_1, c_2 and c_3 such that the following estimators

$$\hat{\theta}_1 = c_1 Y_1, \hat{\theta} = c_2 Y_2 \text{ and } \hat{\theta}_3 = c_3 \max(Y_1, Y_2),$$

are unbiased. (*Hint:* For $\hat{\theta}_3$, integrate the function $\max(y_1, y_2)$ multiplied by the joint density of Y_1, Y_2. Split the integral over $[0, \theta] \times [0, \theta]$ into two parts - one where $y_1 \geq y_2$ and the other where $y_1 < y_2$ and note that $\max(y_1, y_2) = y_1 1_{\{y_1 \geq y_2\}} + y_2 1_{\{y_1 < y_2\}}$.)

ii. $(2 + 3 + 10 = 15$ points) With values c_1, c_2 and c_3 as above, compute mean-squared errors MSE $\left(\hat{\theta}_1\right)$, MSE $\left(\hat{\theta}_2\right)$ and MSE $\left(\hat{\theta}_3\right)$ of $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_3$.

iii. (10 points) Sketch the graphs of MSE $\left(\hat{\theta}_1\right)$, MSE $\left(\hat{\theta}_2\right)$ and MSE $\left(\hat{\theta}_3\right)$ as functions of θ. Is one of the three clearly better (in the mean-square sense) than the others?