M362K: March 20th, 2024.

Conditional Distribution.

We are interested in the following type of probabilities:

$$\mathbb{P}[X \in B \mid E] \overset{\text{def'n}}{=} \frac{\mathbb{P}[\{X \in B\} \cap E]}{\mathbb{P}[E]}$$

$B \subseteq \mathbb{R}$ (a region in $\mathbb{R}$)

$E \subseteq \Omega$ (an event)

For a given (temporarily fixed) event E, the above family of probabilities (numbers) defines a distribution on the real line $\mathbb{R}$. (we associate a number (probability) w/ every "nice" region B in $\mathbb{R}$).

For a finite Ω, it suffices to keep track of

$\boxed{\mathbb{P}[X=x \mid E]}$ for all x in the support of X

Then,

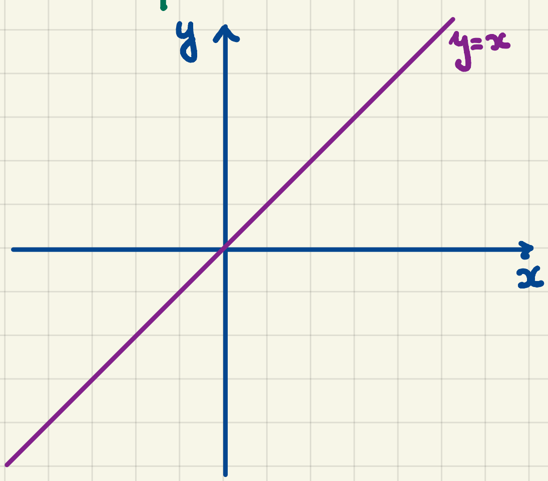
$$\boxed{\mathbb{P}[X \in B \mid E] = \sum_{x \in B} \mathbb{P}[X=x \mid E]}$$

We got the conditional distribution of X given E.

Interesting Events.

We look @ a couple of events determined by the realization of the random pair (X,Y).

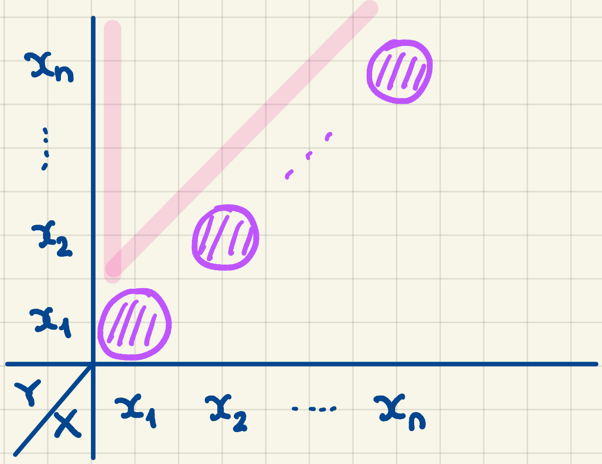
e.g.,



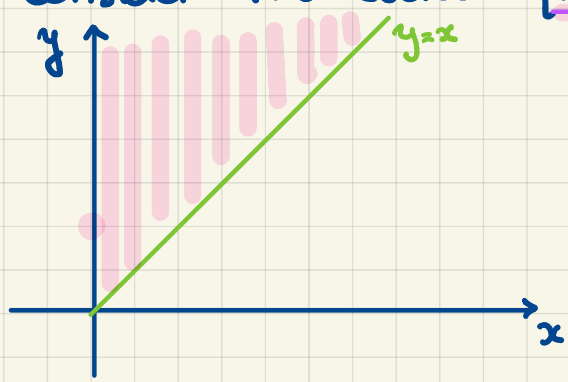
$\{X=Y\}$

$$\mathbb{P}[X=Y] = \sum_{(x,y):x=y} \mathbb{P}[X=x, Y=y]$$

$$\boxed{= \sum_{\text{all } x} p_{X,Y}(x,x)}$$



e.g., Consider the event $\{X<Y\}$



$$\mathbb{P}[X<Y] = \sum_{(x,y):\, x<y} p_{X,Y}(x,y) = \sum_{\text{all } x} \sum_{y>x} p_{X,Y}(x,y)$$

Example 3.1.3.

We extract @ random and without replacement two slips of paper out of a box containing three slips numbered 1, 2, 3.

X... the # on first slip
Y... the # of second slip

Y \ X	1	2	3
3	1/6	1/6	0
2	1/6	0	1/6
1	0	1/6	1/6

Define: $\boxed{S = X+Y}$ Support(S) = $\{3, 4, 5\}$

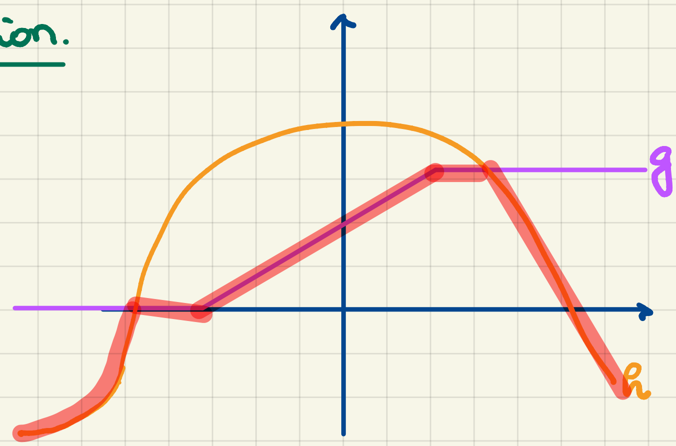
possible values	3	4	5
probab.s	1/3	1/3	1/3

The general formula for the sum $S = X+Y$ of the random variables w/ the joint pmf $p_{X,Y}(\cdot,\cdot)$ is

$$p_S(s) = \mathbb{P}[S=s] = \mathbb{P}[X+Y=s] = \sum_{(x,y):x+y=s} p_{X,Y}(x,y) = \sum_{\text{all } x} p_{X,Y}(x, s-x)$$

$$= \sum_{\text{all } y} p_{X,Y}(s-y, y)$$

Inspiration.



Q: What is the minimum of functions g and h?

Example. You're waiting for the bus @ the bus stop. There are two bus lines you can take: A and B. You take whichever bus arrives first.

X... waiting time (in min) until A arrives
Y... waiting time (in min) until B arrives

Y \ X	1	2	3
2	3/10	1/5	1/10
1	1/10	1/5	1/10

T... waiting time until you get on a bus

$\boxed{T = \min(X,Y)}$ Support$(T) = \{1, 2\}$

possible values	1	2
probabs	7/10	3/10