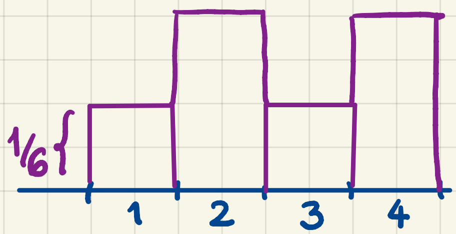
M362K: January 31st, 2024.

**Example B.** Consider a box w/ numbered balls in it.
Let there be 3 balls w/ #1,
6 balls w/ #2,
3 balls w/ #3,
6 balls w/ #4.

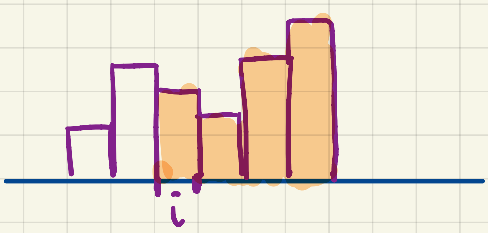
A ball is chosen @ random.
Let's draw the histogram of the distribution in this experiment

Elemantary outcomes: 1, 2, 3, 4
The dist'n

| Result | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Prob. | 1/6 | 1/3 | 1/6 | 1/3 |




Q:




What does the probability that the obsewed number is greater than or equal to $i$ correspond to in the histogram?

Take the area of the shaded region and divide by the area under the entire histogram.

**Note.** We got the same distribution from two different experiments. We say that the two procedures are probabilistically equivalent.

# Named Distributions.

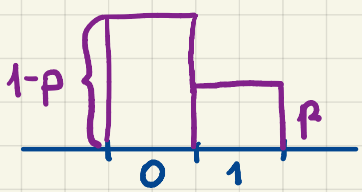
* See Appendix of text
* Most of them have parameters which are subject to constraints (like nonnegativity)

## Bernoulli

Outcome space: $\Omega = \{0, 1\}$

$p$... probability of outcome 1, i.e., $\mathbb{P}[\{1\}] = p$

our parameter in this situation w/ a constraint $\boxed{p \in [0,1]}$

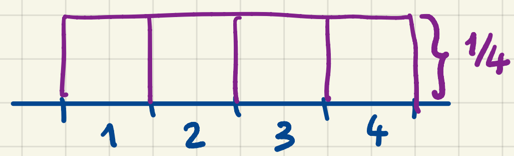



## Uniform distribution on a finite set

$\Omega$ ... finite outcome space

$\#(\Omega) = n$

equally likely outcomes w/ prob $= \frac{1}{n}$




---

## Example.

$\mathbb{P}[E] = 11.16\%$
$\mathbb{P}[A] = 8.50\%$
$\mathbb{P}[I] = 7.54\%$
$\mathbb{P}[O] = 7.16\%$
$\mathbb{P}[U] = 3.63\%$

$$\mathbb{P}[\text{a consonant is chosen}] = 1 - 0.1116 - 0.085 - 0.0754 - 0.0716 - 0.0363$$
$$= \dots$$

**Problem.** Let $A$ and $B$ be two events on the same outcome space.
You know that
$$\mathbb{P}[A] = 1/5 \quad \text{and} \quad \mathbb{P}[B] = 2/5 .$$

Find the probability that @ least one of the events $A$ and $B$ occurs, if

(i) $\mathbb{P}[A \cap B] = 0.15$

$\longrightarrow$: $\mathbb{P}[A \cup B] = \mathbb{P}[A] + \mathbb{P}[B] - \mathbb{P}[A \cap B] = 0.2 + 0.4 - 0.15 = 0.45$ $\square$

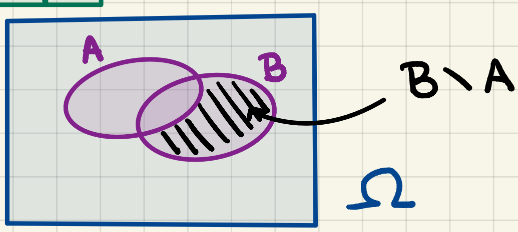
(ii) $A$ and $B$ are mutually exclusive

$\longrightarrow$: $\mathbb{P}[A \cup B] = \mathbb{P}[A] + \mathbb{P}[B] = 0.6$ $\square$

(iii) If $A$, then $B$. $\Longleftrightarrow$ $A \subseteq B$

$\longrightarrow$: $\mathbb{P}[A \cup B] = \mathbb{P}[B] = 0.4$ $\square$

(iv) $\boxed{\text{Def'n.}}$




$\boxed{\mathbb{P}[B \setminus A] = 0.35}$

mutually exclusive

$$\mathbb{P}[A \cup B] = \mathbb{P}[A \cup (B \setminus A)]$$
$$= \mathbb{P}[A] + \mathbb{P}[B \setminus A]$$
$$= 0.2 + 0.35 = 0.55 \quad \square$$

**Problem.** A class contains 10 men and 20 women.
Half the men and half the women have brown eyes.
Find the probability that a randomly chosen person in a man or has brown eyes.

$\longrightarrow$: $M = \{\text{person is man}\}$

$B = \{\text{person has brown eyes}\}$

$$\mathbb{P}[M \cup B] = \mathbb{P}[M] + \mathbb{P}[B] - \mathbb{P}[M \cap B] = \frac{10}{30} + \frac{15}{30} - \frac{5}{30} = \frac{20}{30} = \frac{2}{3} \quad \square$$

Inclusion Exclusion