M362K: January 29th, 2024.

De Méré's Paradox.

Let Ω_1 be the outcome space for 4 rolls of a single die.

Then, $\Omega_1 = \{(a_1, a_2, a_3, a_4) : 1 \le a_i \le 6, \ i = 1,2,3,4\}$

and $\#(\Omega_1) = \underline{6} \cdot \underline{6} \cdot \underline{6} \cdot \underline{6} = 6^4$

Let A be the event that @ least one roll is a six.

$$\#(A) = \#(\Omega) - \#(A^c) = 6^4 - 5^4$$

$$\#(A^c) = \underline{5^4}$$

Finally, $\mathbb{P}[A] = 1 - \left(\frac{5}{6}\right)^4$

Let Ω_2 be the outcome space for 24 throws of a pair of dice.

$$\#(\Omega_2) = \underline{36^{24}}$$

Let B be the event that @ least one double 6 appears in the 24 throws.

$$\#(B) = \underline{36^{24} - 35^{24}}$$

So, $\mathbb{P}[B] = 1 - \left(\frac{35}{36}\right)^{24}$

Note: $\left(\frac{35}{36}\right)^{24} > \left(\frac{5}{6}\right)^4$ So: $\boxed{\mathbb{P}[B] < \mathbb{P}[A]}$

$\square$

Problem. Let $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and let $\mathbb{P}$ be a distribution over Ω.

Assume:

$$\mathbb{P}[\{\omega_1\}] = \frac{1}{3}, \ \mathbb{P}[\{\omega_2\}] = \frac{1}{6}, \text{ and } \mathbb{P}[\{\omega_3\}] = \frac{1}{9}.$$

Find

$$\mathbb{P}[\{\omega_4\}] = ?$$

$\longrightarrow$:

$$1 = \frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \mathbb{P}[\{\omega_4\}]$$

answer:

$$1 - \underbrace{\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{9}\right)} = 1 - \frac{6+3+2}{18} = 1 - \frac{11}{18} = \frac{7}{18}$$

$\square$

Problem. Let $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ be an outcome space and let $\mathbb{P}$ is a distribution over Ω.

Assume:

$$\mathbb{P}[\{\omega_3\}] = \mathbb{P}[\{\omega_4\}] = \frac{1}{4}$$

$$\text{and } \mathbb{P}[\{\omega_1\}] = 2\,\mathbb{P}[\{\omega_2\}].$$

$\mathbb{P}[\{\omega_2\}] = x$

Find $\mathbb{P}[\{\omega_1\}]$.

$\longrightarrow$:

$$2x + x + \frac{1}{4} + \frac{1}{4} = 1$$

$$3x = \frac{1}{2}$$

$$x = \frac{1}{6} \quad \Rightarrow \quad \text{answer: } \boxed{\frac{1}{3}}$$

$\square$

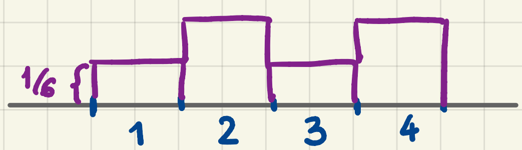
Histograms.

Example A. Consider a loaded tetrahedron w/ vertices labeled 1, 2, 3, 4 such that the faces w/ even numbers are twice as likely as the faces w/ odd numbers.
Let p_i be the probability of result $i = 1, 2, 3, 4$.

Then, $\left.\begin{array}{l} x = p_1 = p_3 \\ 2x = p_2 = p_4 \end{array}\right\}$ $\quad x + 2x + x + 2x = 1 \Rightarrow 6x = 1$

$\Rightarrow x = \frac{1}{6}$

Results	1	2	3	4
Probab.	$1/6$	$1/3$	$1/6$	$1/3$



Histogram is a graph describing a distribution where the height of the bar above a value is proportional to the probability of that value.