M362K: January 19th, 2024.

Reading Assignment: Appendices 1-4.

Section 1.1. Inspiration

"Equally likely outcomes"

(aka "picked/chosen @ random")

Ω ... set of all possible elementary outcomes ω

Note: $\omega \in \Omega$

Start w/ finite for simplicity.

e.g., cointoss $\Omega = \{H, T\}$; $\#(\Omega) = 2$;

roll of a die $\Omega = \{1, 2, 3, 4, 5, 6\}$; $\#(\Omega) = 6$;

rock·paper·scissors $\Omega = \{R, P, S\} \times \{R, P, S\}$

$= \{(R,R), (R,P), (R,S),$
$(P,R), (P,P), (P,S),$
$(S,R), (S,P), (S,S)\}$; $\#(\Omega) = 9 = 3 \times 3$

fastcasual restaurant:

$\Omega = \{\text{rice}, \text{pasta}\} \times \{\text{tofu}, \text{salmon}, \text{chicken}\} \times \{\text{carrots}, \text{peas}, \text{beans}\}$

$\#(\Omega) = 2 \times 3 \times 3 = 18$

Any "nice" subset of Ω is called an event;

e.g., a prime number on the die $A = \{2, 3, 5\}$; $\#(A) = 3$

the first player wins $B = \{(R,S), (P,R), (S,P)\}$;

vegeterian meals: $E = \{\text{rice}, \text{pasta}\} \times \{\text{tofu}\} \times \{\text{carrots}, \text{peas}, \text{beans}\}$ $\#(E) = 6$

meals w/ beans: $F = \{\text{rice}, \text{pasta}\} \times \{\text{tofu}, \text{salmon}, \text{chicken}\} \times \{\text{beans}\}$

veg meals w/ beans: $G = \{\text{rice}, \text{pasta}\} \times \{\text{tofu}\} \times \{\text{beans}\}$

$= E \cap F$

If all outcomes are equally likely, that's the same as if we said they have the same probability/chance/likelihood.

It's natural to have $\mathbb{P}[\Omega] = 1$.

So,

$$\mathbb{P}\left[\{\omega\}\right] = \frac{1}{\#(\Omega)} \quad \text{for all } \omega$$

singleton

In fact, for every event $E \subseteq \Omega$, it's natural to set

$$\mathbb{P}[E] = \frac{\#(E)}{\#(\Omega)}$$

e.g., $\mathbb{P}[\text{prime number on die}] = \frac{3}{6} = \frac{1}{2}$

$\mathbb{P}[\text{veg. meal}] = \frac{\cancel{2} \times 1 \times \cancel{3}}{\cancel{2} \times 3 \times \cancel{3}} = \frac{1}{3}$

Example. [Example 3]

You roll two n-sided dice w/ sides numbered $1, 2, 3, \ldots, n$. (w/ $n \geq 4$).

Q: What's the probability that the sum is 5?

$\longrightarrow$: $\Omega = \{(i,j) : 1 \leq i,j \leq n\}$

$\#(\Omega) = n \times n = n^2$

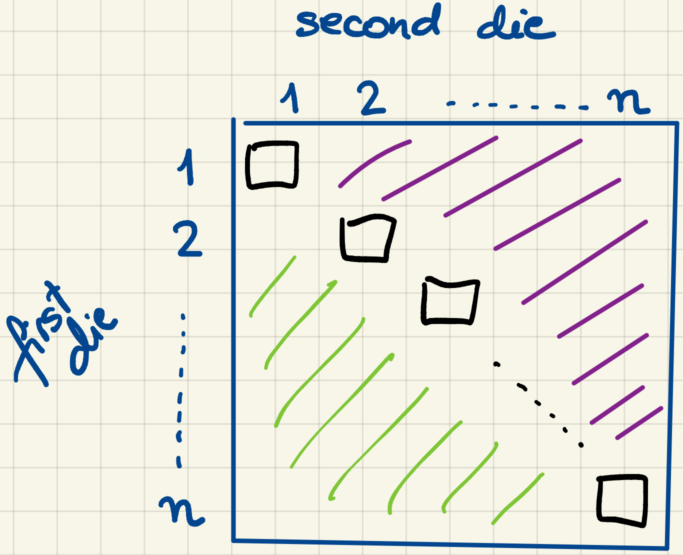
$H = \{\text{sum is } 5\} = \{(1,4), (2,3), (3,2), (4,1)\}$

$$\mathbb{P}[H] = \frac{\#(H)}{\#(\Omega)} = \frac{4}{n^2}$$

$\square$

Q: What's the probability that the number on the second die is strictly greater?

$\longrightarrow$:



$D = \{\text{same on both dice}\}$ $\qquad \boxed{\#(D) = n}$

$E = \{\text{second greater}\}$

$C = \{\text{first greater}\}$ $\qquad$ symmetry $\#(E) = \#(C)$

$$\#(\Omega) = \underline{\#(E)} + \#(D) + \underline{\#(C)} = 2\#(E) + \#(D) \Rightarrow \#(E) = \frac{\overbrace{\#(\Omega)}^{=n^2} - \#(D)}{2}$$

$$\mathbb{P}[E] = \frac{\#(E)}{\#(\Omega)} = \frac{\frac{1}{2}(n^2 - n)}{n^2} = \frac{1}{2}\left(1 - \frac{1}{n}\right) \xrightarrow[n\to\infty]{} \frac{1}{2}$$

$$\frac{1}{n} \xrightarrow{n\to\infty} 0$$

Problem. Balls in an urn: Y, B, R, G.

Twice as many Y as B. $\Rightarrow$ $y = 2b = 8g$
Twice as many B as R. $\Rightarrow$ $b = 2r = 4g$
Twice as many R as G. $\Rightarrow$ $r = 2g$

$\mathbb{P}[G] = ?$

$\rightarrow$: y, b, r, g

$1 = g + 2g + 4g + 8g$

$\boxed{g = \frac{1}{15}}$ $\square$