M362K: February 21st, 2024.

26. A study of automobile accidents produced the following data:

Model year	Proportion of all vehicles	Probability of involvement in an accident
2014	0.16	0.05
2013	0.18	0.02
2012	0.20	0.03
Other	0.46	0.04

An automobile from one of the model years 2014, 2013, and 2012 was involved in an accident.

Calculate the probability that the model year of this automobile is 2014.

(A) 0.22
(B) 0.30
(C) 0.33
(D) 0.45
(E) 0.50

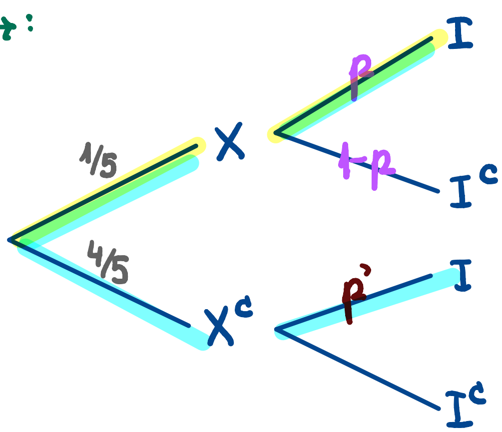
27. A hospital receives 1/5 of its flu vaccine shipments from Company X and the remainder of its shipments from other companies. Each shipment contains a very large number of vaccine vials.

For Company X's shipments, 10% of the vials are ineffective. For every other company, 2% of the vials are ineffective. The hospital tests 30 randomly selected vials from a shipment and finds that one vial is ineffective.

Calculate the probability that this shipment came from Company X.

(A) 0.10
(B) 0.14
(C) 0.37
(D) 0.63
(E) 0.86

I = {one vial out of 30 was ineffective}



$p = \binom{30}{1}(0.1)(0.9)^{29} = 0.1413$

$p' = \binom{30}{1}(0.02)(0.98)^{29} = 0.334$

$$\mathbb{P}[X \mid I] = \frac{\mathbb{P}[X \cap I]}{\mathbb{P}[I]} = \frac{0.2 \cdot 0.1413}{0.2 \cdot 0.1413 + 0.8 \cdot 0.334} = 0.0956576$$

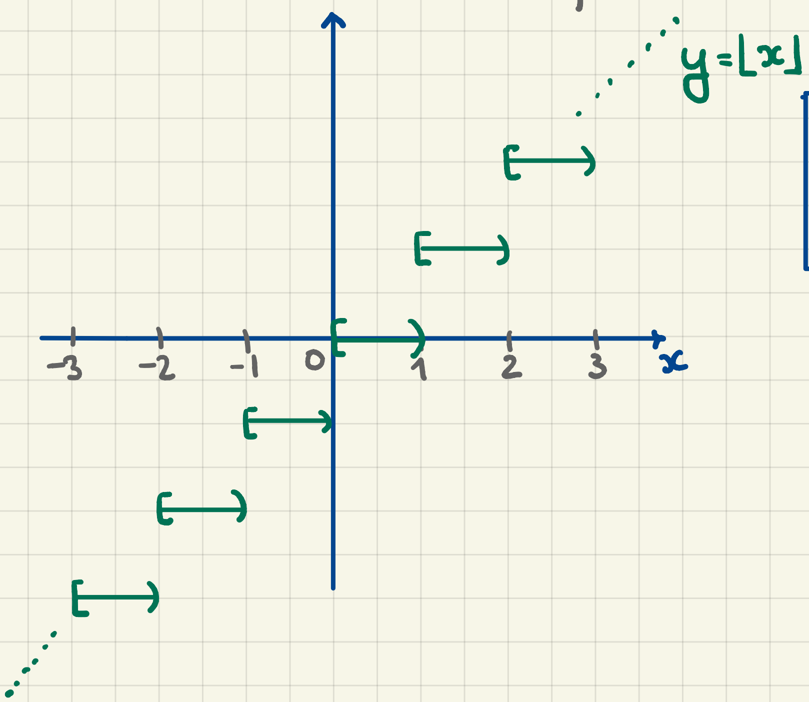
$\square$

Mode of the Binomial Dist'n.

Consider Binomial(n, p).

Then, the most likely number of successes, i.e., the mode of the distribution is $\boxed{m = \lfloor n \cdot p + p \rfloor}$

Floor function (integer part function) assigns to each real number the largest integer that's less than or equal to that number.



This is an example of a step function.

If $n \cdot p + p$ is an integer, then the modes are m and $m-1$.

Problem 2.1.8 from Pitman.

For each positive integer n, what is the largest value of p such that zero is the most likely number of successes in n independent Bernoulli trials w/ success probability p?

$\longrightarrow$: From our formula for the mode, we will have

$$0 = m = \lfloor np + p \rfloor .$$

Then,
$$np + p < 1$$
$$(n+1)p < 1$$
$$\boxed{p < \frac{1}{n+1}}$$

Alternatively, If $np + p = 1$ we have modes @ 0 and 1 and we have $\boxed{p = \frac{1}{n+1}}$

$\square$

Standard Normal Distribution.

Here, $\Omega = \mathbb{R}$, and we will look @ a family of probability distributions on $\Omega = \mathbb{R}$.

The standard normal density function is given by

$$\boxed{\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \qquad \text{for all } z \in \mathbb{R}}$$

The curve $y = \varphi(z)$ is called the standard normal curve.

Properties:

- $\varphi(z) > 0$ for all $z \in \mathbb{R}$
- φ is $\boxed{\text{even}}$, i.e., it's symmetric about the vertical axis, i.e., $\boxed{\varphi(z) = \varphi(-z)}$
- $\boxed{\int_{-\infty}^{\infty} \varphi(z)\, dz = 1}$
- changes from convex to concave to convex at -1 and $+1$.

If $\mathbb{P}$ stands for the standard normal distribution on $\Omega = \mathbb{R}$, then, for all $a < b$

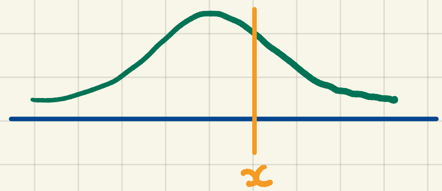
$$\boxed{\mathbb{P}[(a,b]] = \int_a^b \varphi(z)\,dz}$$

e.g., $\mathbb{P}[(-1,1]] \approx 0.68$

$\mathbb{P}[(-2,2]] \approx 0.95$

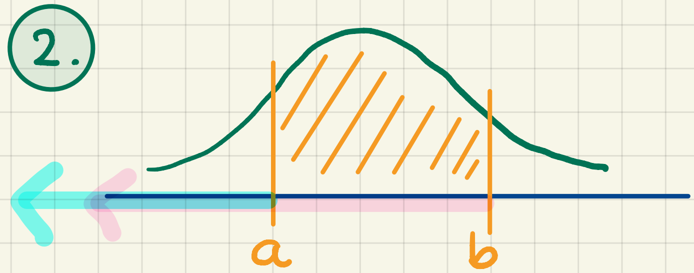
$\mathbb{P}[(-3,3]] \approx 0.9973$

Note: 1.



$$\boxed{\mathbb{P}[\{x\}] = 0}$$

2.

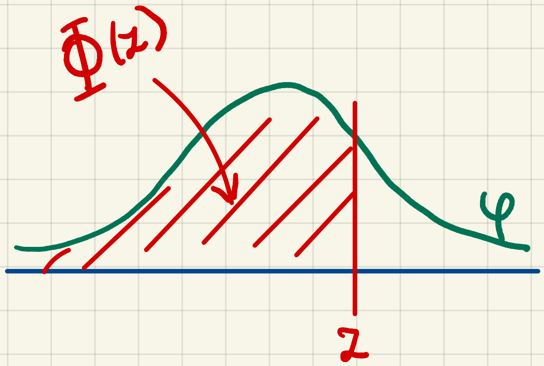


$\mathbb{P}[(a,b]] = \mathbb{P}[(-\infty,b]] - \mathbb{P}[(-\infty,a]]$

So, it's sufficient to look @ $\boxed{\mathbb{P}[(-\infty,x]]}$

The standard normal cumulative distribution function is

$$\boxed{\Phi(z) = \int_{-\infty}^{z} \underline{\varphi(u)}\,du \quad \text{for all } z \in \mathbb{R}}$$



Properties:

- Φ is strictly increasing
- $\lim_{z \to -\infty} \Phi(z) = \underline{0}$
- $\lim_{z \to \infty} \Phi(z) = 1$