M362K: February 9th, 2024.

24. A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not actually present. One percent of the population actually has the disease.

Calculate the probability that a person actually has the disease given that the test indicates the presence of the disease.

(A) 0.324
(B) 0.657
(C) 0.945
(D) 0.950
(E) 0.995

25. The probability that a randomly chosen male has a blood circulation problem is 0.25. Males who have a blood circulation problem are twice as likely to be smokers as those who do not have a blood circulation problem.

$$\boxed{\mathbb{P}[S \mid C] = 2\cdot \mathbb{P}[S \mid C^c]}$$

Calculate the probability that a male has a blood circulation problem, $\boxed{\text{given}}$ that he is a smoker.

(A) 1/4
(B) 1/3
(C) 2/5
(D) 1/2
(E) 2/3

$\rightarrow$:

$$\mathbb{P}[C \mid S] = \frac{\mathbb{P}[C\cap S]}{\mathbb{P}[S]} = \frac{\overbrace{\mathbb{P}[S \mid C]}^{2\boxed{\cancel{\mathbb{P}[S|C^c]}}}\cdot \overset{0.25}{\mathbb{P}[C]}}{\underbrace{\mathbb{P}[S \mid C]}_{2\boxed{\cancel{\mathbb{P}[S|C^c]}}}\cdot \underset{0.25}{\mathbb{P}[C]} + \boxed{\cancel{\mathbb{P}[S \mid C^c]}}\cdot \underset{0.75}{\mathbb{P}[C^c]}}$$

$$= \frac{2\cdot\frac{1}{4}}{2\cdot\frac{1}{4}+3\cdot\frac{1}{4}} = \frac{2}{5} \quad \square$$

$\boxed{\mathbb{P}\#24.}$

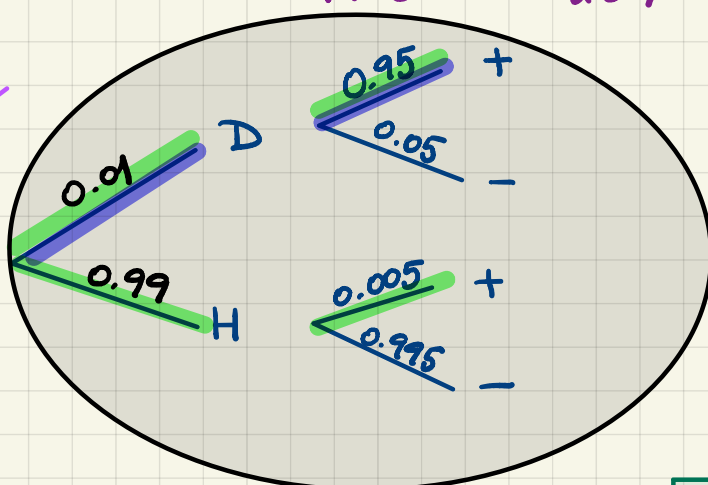
Method I.

	D	H	
+	$0.01\cdot 0.95$	$0.99\cdot 0.005$	$0.01\cdot 0.95 + 0.99\cdot 0.005$
−	$0.01\cdot 0.05$		
	0.01	0.99	1

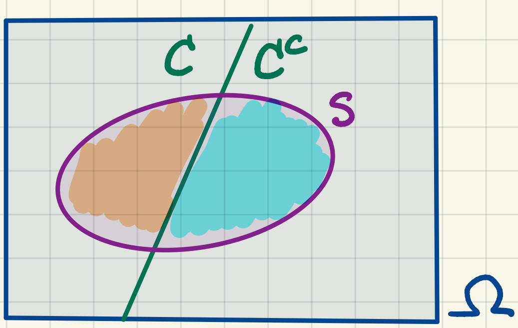
$$\mathbb{P}[D \mid +] = \frac{\mathbb{P}[D\cap +]}{\mathbb{P}[+]} = \frac{0.01\cdot 0.95}{0.01\cdot 0.95 + 0.99\cdot 0.005} = \frac{95}{95+49.5}$$

$$= \frac{950}{1445} = \frac{190}{289} \quad \square$$

Method II.



$\square$



$$\mathbb{P}[S] = \mathbb{P}[S\cap C] + \mathbb{P}[S\cap C^c]$$

$$= \mathbb{P}[C]\cdot \mathbb{P}[S \mid C] + \mathbb{P}[C^c]\cdot \mathbb{P}[S \mid C^c]$$

26. A study of automobile accidents produced the following data:

Do this @ home

Model year	Proportion of all vehicles	Probability of involvement in an accident
2014	0.16	0.05
2013	0.18	0.02
2012	0.20	0.03
Other	0.46	0.04

An automobile from one of the model years 2014, 2013, and 2012 was involved in an accident.

Calculate the probability that the model year of this automobile is 2014.

(A) 0.22
(B) 0.30
(C) 0.33
(D) 0.45
(E) 0.50

27. A hospital receives 1/5 of its flu vaccine shipments from Company X and the remainder of its shipments from other companies. Each shipment contains a very large number of vaccine vials.

For Company X's shipments, 10% of the vials are ineffective. For every other company, 2% of the vials are ineffective. The hospital tests 30 randomly selected vials from a shipment and finds that one vial is ineffective.

Calculate the probability that this shipment came from Company X.

(A) 0.10
(B) 0.14
(C) 0.37
(D) 0.63
(E) 0.86

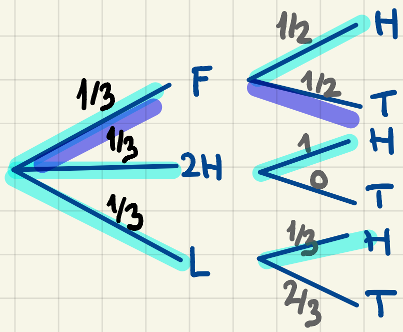
Next week :)

Problem. A box contains three coins: one coin is fair,
one coin has two heads,
one coin is weighted so that the probab. of heads is $1/3$.

A coin is selected @ random and tossed.

(i) Find the probability that the outcome is heads.

$\longrightarrow$:



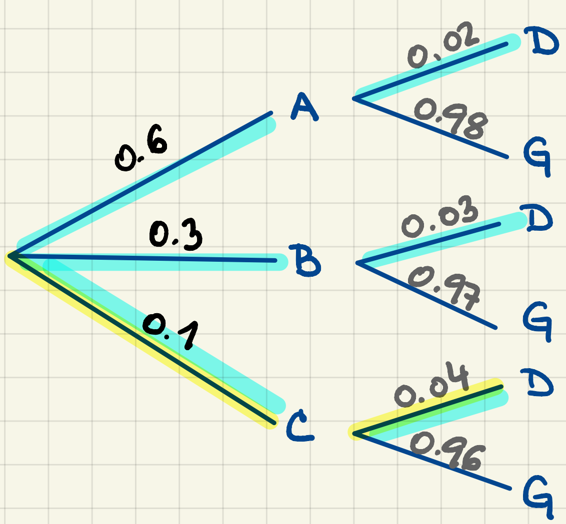
$$\mathbb{P}[H] = \frac{1}{3}\left(\frac{1}{2} + 1 + \frac{1}{3}\right) = \frac{1}{3} \cdot \frac{3+6+2}{6} = \frac{11}{18}$$

(ii) You observe that the result is tails. What's the probab. that the coin was fair?

$\longrightarrow$:
$$\mathbb{P}[F \mid T] = \frac{\mathbb{P}[F \cap T]}{\mathbb{P}[T]} = \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{7}{18}} = \frac{3}{7}$$
$\square$

Problem. Three machines A, B, and C produce, respectively, 60%, 30%, and 10% of the total number of items. The percentages of defective output are 2%, 3%, and 4%, respectively.
An item is selected @ random and found defective. Find the probab. that the item was produced by machine C.

$\longrightarrow$:



$$\mathbb{P}[C \mid D] = \frac{\mathbb{P}[C \cap D]}{\mathbb{P}[D]} =$$

$$= \frac{0.1 \cdot 0.04}{0.6 \cdot 0.02 + 0.3 \cdot 0.03 + 0.1 \cdot 0.04}$$

$$= \frac{4}{12+9+4} = \frac{4}{25}$$
$\square$

Problem. Let A denote the event that a randomly chosen family has children of two genders.
Let B denote the event that a family has @ most one boy.
Assume that the two genders are equally likely and mutually independent and that a family has two children.
Are the events A and B independent?

Def'n. Events E and F are said to be *independent* if

$$\boxed{\mathbb{P}[E \cap F] = \mathbb{P}[E] \cdot \mathbb{P}[F]}$$

$\longrightarrow$:

The outcome space: $\Omega = \{BB, GB, BG, GG\}$

$$\left.\begin{aligned} A &= \{GB, BG\} \\ B &= \{GB, BG, GG\} \end{aligned}\right\} A \cap B = A$$

We need to see if:

$$\mathbb{P}[A \cap B] \overset{?}{=} \mathbb{P}[A] \cdot \mathbb{P}[B]$$

$$\mathbb{P}[A] = \frac{1}{2}$$

$$\mathbb{P}[B] = \frac{3}{4}$$

$$\mathbb{P}[A \cap B] = \frac{1}{2} \neq \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

Not.

$\square$

Problem. Do the above assuming families w/ 3 children!

$\longrightarrow$: $\tilde{\Omega} = \{BBB, GBB, BGB, BBG, GGB, GBG, BGG, GGG\}$

All outcomes are equally likely.

$\tilde{A} = \{GBB, BGB, BBG, GGB, GBG, BGG\}$ $\quad \mathbb{P}[\tilde{A}] = \frac{6}{8} = \frac{3}{4}$

$\tilde{B} = \{GGB, GBG, BGG, GGG\}$ $\quad \mathbb{P}[\tilde{B}] = \frac{4}{8} = \frac{1}{2}$

$\tilde{A} \cap \tilde{B} = \{GGB, GBG, BGG\}$ $\quad \mathbb{P}[\tilde{A} \cap \tilde{B}] = \frac{3}{8}$

They are independent!

$\square$