M362K: February 7th, 2024.

Problem. $\boxed{\text{1.R.7 from Pitman}}$

A population of 50 registered voters votes on Prop134.
We know that

30 are in favor and 20 are opposed.

An opinion survey selects a random sample of 4 voters from the population.

(a) What is the probab. that there will be no one in favor of Prop134 in the sample?

$\longrightarrow$:
$$\frac{20}{50}\cdot\frac{19}{49}\cdot\frac{18}{48}\cdot\frac{17}{47} = 0.021$$
$\square$

(b) What is the probab. that there will be @ least one person in favor?

$\longrightarrow$: $1 - 0.021 = 0.979$

$\square$

(c) What is the probab. that exactly one person is in favor?

$\longrightarrow$: Method I.
$$4\cdot\frac{30}{50}\cdot\frac{20}{49}\cdot\frac{19}{48}\cdot\frac{18}{47}$$

Method II.
$$\frac{\binom{30}{1}\binom{20}{3}}{\binom{50}{4}} = \frac{30\cdot\frac{20\cdot19\cdot18}{\cancel{3!}}}{\frac{50\cdot49\cdot48\cdot47}{\cancel{4!}_{\,4}}}$$
$$= 4\cdot\frac{30\cdot20\cdot19\cdot18}{50\cdot49\cdot48\cdot47}$$
$\square$

(d) What is the probab. that the strict majority of the sample will be FOR Prop134?

$\longrightarrow$: 3 or 4 are FOR

1 or 0 are AGAINST

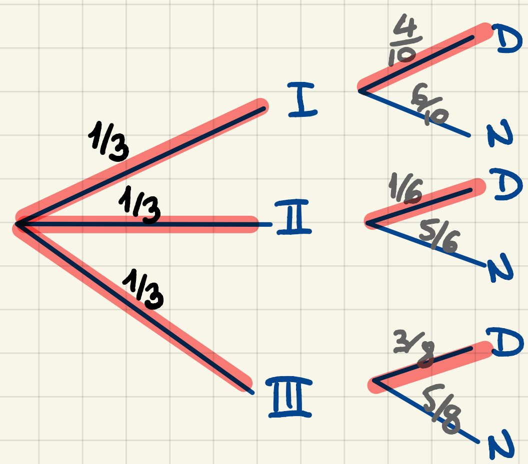
4. $\frac{20}{50} \cdot \frac{30}{49} \cdot \frac{29}{48} \cdot \frac{28}{47} + \frac{30}{50} \cdot \frac{29}{49} \cdot \frac{28}{48} \cdot \frac{27}{47}$ $\square$

Problem. We are given 3 boxes:

- **Box I**: 10 lightbulbs of which 4 defective
- **Box II**: 6 lightbulbs of which 1 defective
- **Box III**: 8 lightbulbs of which 3 defective

We select a box @ random and then draw a bulb at random from that box.
What is the probab. that the lightbulb is defective?

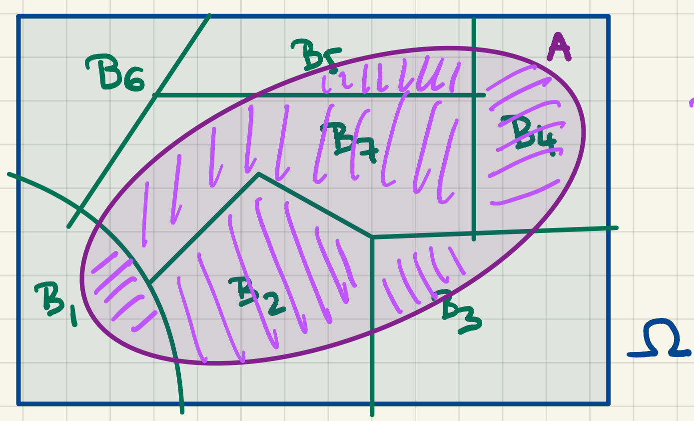
$\longrightarrow$:



$$\frac{1}{3} \cdot \frac{4}{10} + \frac{1}{3} \cdot \frac{1}{6} + \frac{1}{3} \cdot \frac{3}{8} =$$

$$= \frac{1}{3} \cdot \frac{48 + 20 + 45}{120} = \frac{113}{360}$$ $\square$

Rule of Average Conditional Probability
(aka the Law of Total Probability)



Let $B_1, B_2, \dots, B_n$ be a partition of Ω.

$$\mathbb{P}[A] = \mathbb{P}[A \cap B_1] + \mathbb{P}[A \cap B_2] + \dots + \mathbb{P}[A \cap B_n]$$
$$= \mathbb{P}[B_1] \cdot \mathbb{P}[A \mid B_1]$$
$$+ \mathbb{P}[B_2] \cdot \mathbb{P}[A \mid B_2] +$$
$$+ \dots + \mathbb{P}[B_n] \cdot \mathbb{P}[A \mid B_n]$$

Bayes' Rule.

Let $B_1, B_2, \dots, B_n$ be a partition of Ω.
Let A be an event.

$$\mathbb{P}[B_i | A] = \frac{\mathbb{P}[B_i \cap A]}{\mathbb{P}[A]} = \frac{\mathbb{P}[B_i] \cdot \mathbb{P}[A | B_i]}{\mathbb{P}[B_1] \cdot \mathbb{P}[A|B_1] + \dots + \mathbb{P}[B_n] \cdot \mathbb{P}[A|B_n]}$$

Problem. Sample P exam #21.

A health study tracked a group of persons for 5 years.
At the beginning of the study:

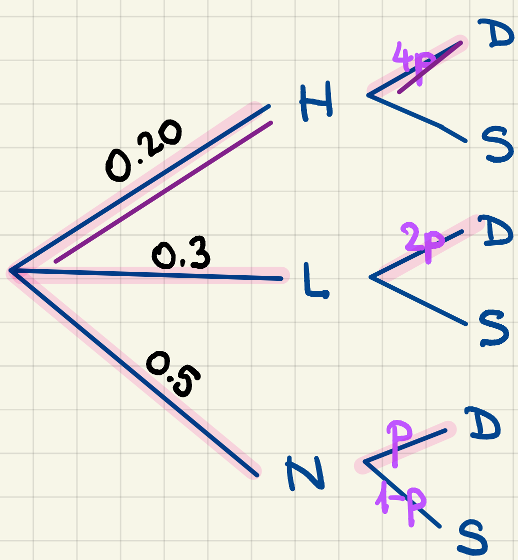
- 20% were classified as heavy smokers;
- 30% were classified as light smokers;
- 50% were nonsmokers.

Results of the study showed that light smokers were twice as likely as nonsmokers to die during the study, but half as likely to die as heavy smokers.

A randomly selected participant died during the study.

Find the probability that they were a heavy smoker.

$\longrightarrow$:



p... the probab. that a non-smoker dies

$$\mathbb{P}[D] = 0.2(4p) + 0.3(2p) + 0.5 \cdot p$$
$$= 1.9p$$

$$\mathbb{P}[HS | D] = \frac{0.2(4\cancel{p})}{1.9\cancel{p}} = \frac{8}{19} \quad \square$$