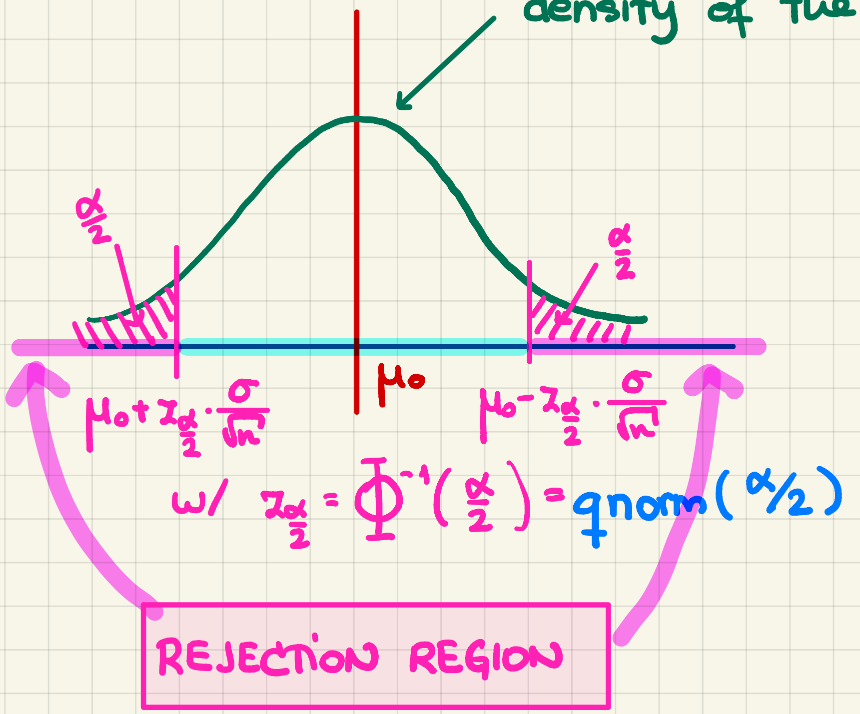
Power of Test.

Temporarily, we focus on the two-sided alternative; we will consider the other two alternatives using analogy.

α ... significance level

density of the sampling dist'n of $\bar{X}$ under the null

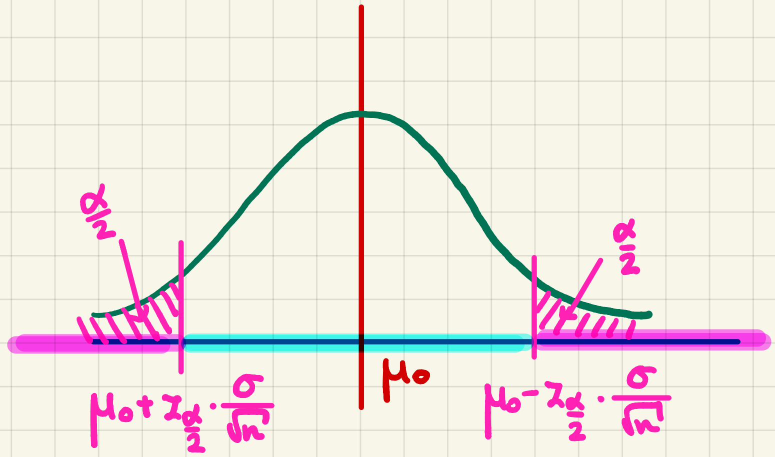


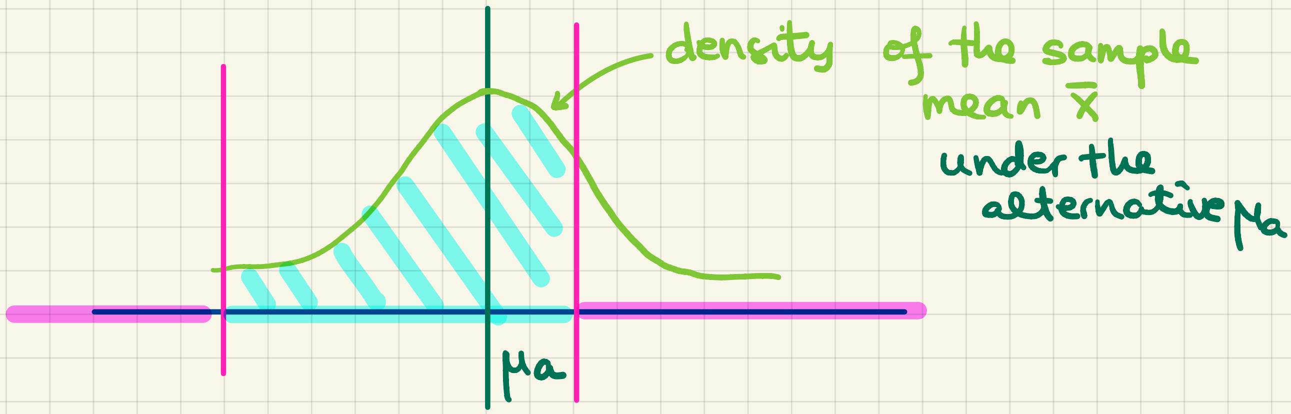
Focus on the "fail-to-reject" region, i.e., for the two-sided test:

$$\left(\mu_0 + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}},\ \mu_0 - z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

Focus on a particular value from the alternative, say μ_a

Q: What is the probability of failing to reject if this particular alternative μ_a is the "true" value of the parameter?





$\mathbb{P}_{\mu_a}$[Fail to Reject H_0] = $\mathbb{P}_{\mu_a}$[Type II Error] $=: \beta$

Note: The smaller the value of β, the BETTER the test!

Def'n. The power of the test @ $\mu = \mu_a$ is defined as $1-\beta$.

Problem. A simple random sample of size 36 is gathered from a normal population w/ an unknown mean μ and the standard deviation of 3.

We are testing:

$$\boxed{H_0 : \mu = 15} \quad \text{vs.} \quad H_a : \mu > 15.$$

The significance level is $\alpha = 0.05$.

Find the power of the test @ $\mu_a = 16$.

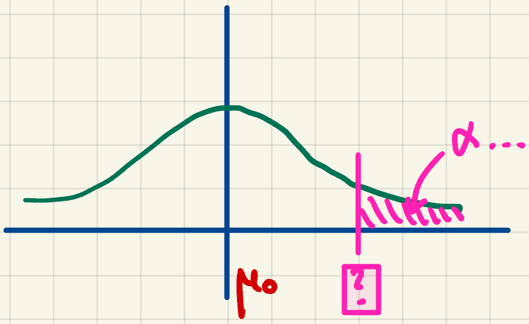
→: First, find the rejection region (RR). ✓

Second, calculate the probability that the sample mean $\bar{X}$ falls into the RR if $\mu = \mu_a = 16$.

↓

Right-tailed test ⇒ RR in raw units is of the form

$$[\,\boxed{?}\,, +\infty)$$



$$\underline{\mu_0 + z_{1-\alpha} \cdot \frac{\sigma}{\sqrt{n}}} \qquad \text{w/ } z_{1-\alpha} = \Phi^{-1}(1-\alpha) = \text{qnorm}(1-\alpha)$$

In this problem, the lower bound of the RR is:

$$\underbrace{15 + (1.645)\frac{3}{\sqrt{36}}}_{0.8225} = \underline{15.8225}$$

$$\boxed{RR = [15.8225, +\infty)}$$

Second, $\underline{\mathbb{P}_{\mu_a}[\bar{X} \geq 15.8225]}$ w/ $\mu_a = 16$

Under this particular alternative, the distribution of the sample mean is

$$\bar{X} \sim \text{Normal}\left(\mu = \mu_a = 16,\ sd = \frac{3}{\sqrt{36}} = 0.5\right)$$

Method I.

1-pnorm (15.8225, mean=16, sd=0.5)= 0.6387052

Method II.

1-pnorm (15+qnorm(0.95)*3/sqrt(36), 16, 0.5) = 0.63876

Method III.

Standardize.

$$\mathbb{P}_{\mu_a}\left[\frac{\bar{X}-16}{0.5} \geq \frac{15.8225-16}{0.5}\right] = \mathbb{P}[Z \geq -0.355] = 0.6406$$

$\sim N(0,1)$

↑ std normal tables

□

University of Texas at Austin

Problem Set # 14

Power of Test.

Provide your **complete solution** for the following problems.

Problem 14.1. As the sample size increases, the power of a test will increase. *True or false? Why?*

→: As a proof of concept: The left-sided alternative.

$H_0: \mu = \mu_0$ vs. $H_a: \mu < \mu_0$