

M358K: October 16th, 2023.

Test of Significance.

Set α ... significance level

Typically: $\alpha = \boxed{0.05}, 0.01, 0.10$

Decision Process.

If $p\text{-value} \leq \alpha$, we **REJECT** the null hypothesis.

If **not**, we **FAIL TO REJECT** the null hypothesis.

Note: The $p\text{-value}$ corresponding to an observed value of the test statistic is the **LOWEST** significance level @ which the null hypothesis would still be **REJECTED**.

Given a **significance level** α , we construct (ahead of data gathering) a **REJECTION REGION (RR)** for our test.

The Null Hypothesis: $H_0: \mu = \mu_0$

The Test Statistic: $\bar{X}$ (the sample mean)

$$\bar{X} \sim \text{Normal}(\text{mean} = \mu, \text{sd} = \frac{\sigma}{\sqrt{n}})$$

The observed value of TS is $\bar{x}$

Its $z\text{-score}$ (under the null):

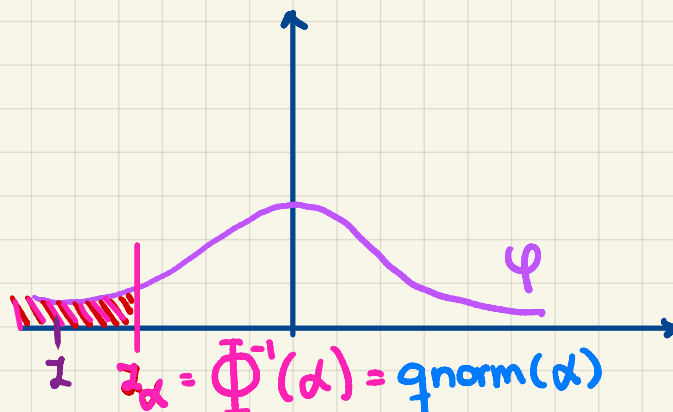
$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

The left-sided alternative:

$$H_a: \mu < \mu_0$$

In standard units:

$$RR = (-\infty, z_\alpha]$$



In raw units:

$$z \leq z_\alpha$$
$$\frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \leq z_\alpha$$

$$\bar{x} \leq \mu_0 + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$$

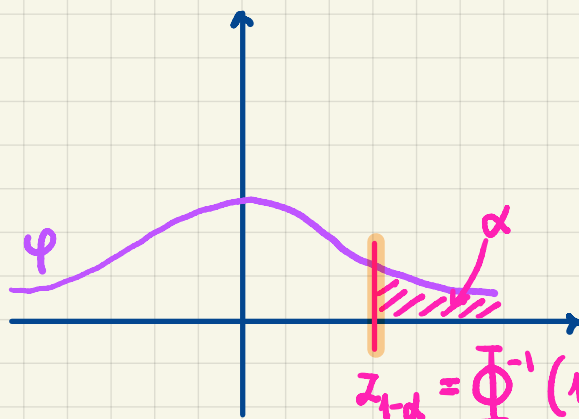
upper bound of the
RR in raw units

$$RR = (-\infty, \mu_0 + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}]$$

The right-sided alternative:

$$H_a: \mu > \mu_0$$

In standard units:



$$z_{1-\alpha} = \Phi^{-1}(1-\alpha) = qnorm(1-\alpha)$$

$$RR = [z_{1-\alpha}, +\infty)$$

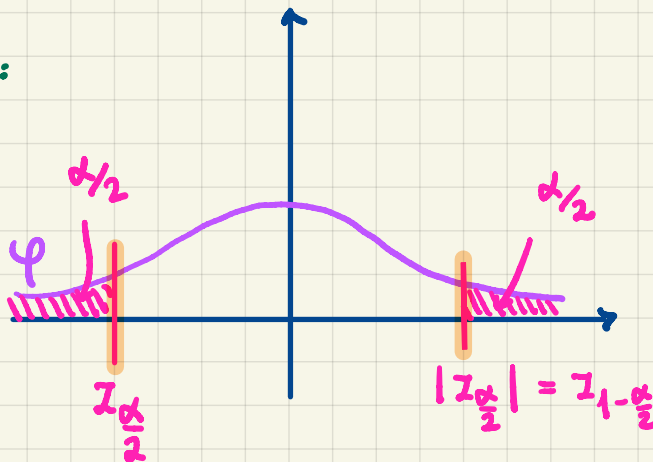
In raw units:

$$RR = [\mu_0 + z_{1-\alpha} \cdot \frac{\sigma}{\sqrt{n}}, +\infty)$$

The Two-Sided Alternative.

$$H_a: \mu \neq \mu_0$$

In standard units:



$$|z_{\frac{\alpha}{2}}| = z_{1-\frac{\alpha}{2}}$$

$$\Phi^{-1}\left(\frac{\alpha}{2}\right) = qnorm(\alpha/2)$$

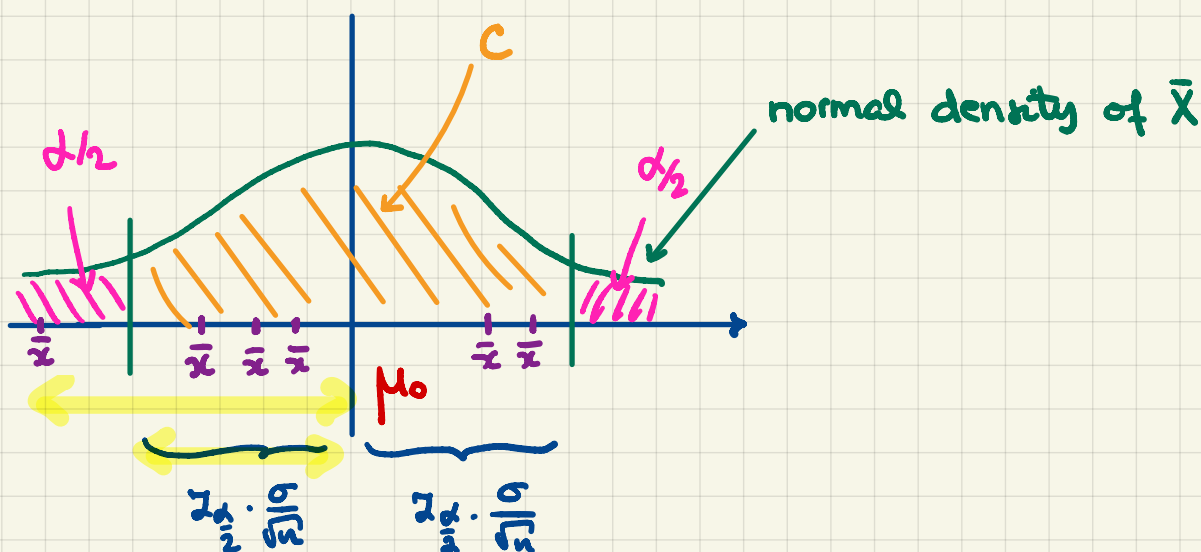
$$RR = (-\infty, -z_{\alpha/2}] \cup [z_{\alpha/2}, +\infty)$$

In raw units:

$$RR = (-\infty, \mu_0 + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}] \cup [\mu_0 + |z_{\alpha/2}| \cdot \frac{\sigma}{\sqrt{n}}, +\infty)$$

Note: There is a relationship between:

- C-confidence intervals
- and
- a two-sided test of significance w/ $\alpha = 1 - C$
(or $C = 1 - \alpha$)



Reject the null $\Leftrightarrow$ the confidence interval does not contain μ_0

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Problem Set # 11

Test of significance.

Problem 11.1. A test of significance can be used to test differences in categorical data. *True or false? Why?*

Problem 11.2. Confidence intervals and two-sided significance tests are linked in the sense that a two-sided test at a significance level α can be carried out in the form of a confidence interval with confidence level $1 - \alpha$. *True or false?*

Problem 11.3. In a test of statistical hypotheses, what does the p -value tell us?

- a. If the null hypothesis is true.
- b. If the alternative hypothesis is true.
- c. The largest level of significance at which the null hypothesis can be rejected.
- d. The smallest level of significance at which the null hypothesis can be rejected

Complete the following statements:

Problem 11.4. When computing p -values, if the p -value is smaller than the chosen significance level α , we say that the results are **STATISTICALLY SIGNIFICANT**.

Provide your complete solution for the following problems.

Problem 11.5. You perform 2000 significance tests using a significance level 0.10. Under the assumption that all of the null hypotheses for the 2000 significance tests are true, how many of the 2000 significance tests would you expect to be statistically significant?

- a. 200
- b. 1800
- c. 2000
- d. 0
- e. None of the above.