M358K : November 17th, 2023.

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Problem Set # 19

The t-procedure.

Provide your **complete solution** for the following problems.

Problem 19.1. When examining data to determine if a $t-$procedure can be used, which of the following are useful?

(a) Histogram
(b) $Q-Q$ plot
(c) Boxplot
(d) All of the above.
(e) None of the above.

Problem 19.2. A simple random sample of 20 third-grade children from a certain school district is selected. Each child is given a test to measure his/her reading ability. You are interested in calculating a 95% confidence interval for the population mean score based on this SRS. The sample mean score is 64 points and the sample standard deviation is 12 points. What is the margin of error associated with the 95% confidence interval? ✓

Problem 19.3. The hypotheses $H_0 : \mu = 10$ versus $\boxed{H_a : \mu \neq 10}$ are examined using a sample of size $n = 18$. The one-sample $t-$statistic has the value of $\boxed{t = -2.05}$. Between what two values does the P-value of this test fall?

(a) $0.01 < P-\text{value} < 0.02$,
(b) $0.02 < P-\text{value} < 0.025$,
(c) $0.025 < P-\text{value} < 0.05$,
(d) $0.05 < P-\text{value} < 0.10$
(e) None of the above.

Problem 19.4. The time (in seconds) needed to clear a certain *Super Mario* run is normally distributed with an unknown mean μ. The local *Bowser basher* league reports the mean clearance time of at most 75 seconds. You are incredulous of their statements and decide to test your hypothesis.

i. What are your null and alternative hypotheses?
ii. Note that the standard deviation of the clearance time is not specified. Which test statistic are you going to use to test your hypotheses?
iii. Let the sample size be 20. What is the distribution of your test statistic?
iv. You gather the data, calculate $\bar{x}$ and s and evaluate the $t-$statistic. The value you obtain equals 1.68. Is your null hypotheses rejected at the siginificance level 0.05?

i. $H_0: \mu = 75$ vs. $\boxed{H_a: \mu > 75}$

ii. $T = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} \sim t(df = 20-1 = 19)$ $\uparrow$ $n=20$

iv. $t = 1.68$

In R: 1-pt(1.68, df=19) = 0.05466274 is the p-value

w/ the significance level of 0.05, we $\boxed{\text{Fail to Reject}}$

Problem#2:

$n=20$; $\bar{x}=64$, $s=12$

margin·of·error = critical value · stderror

based on the t·dist'n w/ $n=20-1=19$ degrees of freedom

confidence level $C=0.95$

$t^*(df=19) = qt((1+0.95)/2, df=19) = 2.093024$

$\text{stderror} = \frac{s}{\sqrt{n}} = 2.683282$

margin·of·error = 5.616173

For Fun: $\boxed{\mu = 56 \pm 5.616173}$

Problem#3:

Method I: t·tables.

$df = 18-1 = 17$

The upper·tail probability corresponding to 2.05 is between 0.025 and 0.05.

This is a two·tailed test, the p·value is between 0.05 and 0.10.

Method II: R.

2*pt(-2.05, df=17) = 0.05611492

Problem. A medicine dispensing machine is supposed to dispense 20 ml of medication. Of course, the amount dispensed is not exact. You model the actual amount as normal w/ std deviation of 1.5 ml.
A sample of 9 doses is collected and measured.
With a particular significance level, the rejection region is the complement of $(19.1, 20.9)$.

What is the power of the test @ $\mu_a = 21.5$?

$\rightarrow$: Under the alternative $\mu_a = 21.5$,

$$\bar{X} \sim \text{Normal}\left(\text{mean} = 21.5,\ \text{sd} = \frac{1.5}{\sqrt{9}} = 0.5\right)$$

$$\mathbb{P}_{\mu_a}[\text{Type II Error}] = \mathbb{P}_{\mu_a}[19.1 < \bar{X} < 20.9]$$

pnorm(20.9, 21.5, 0.5) - pnorm(19.1, 21.5, 0.5)

= 0.1150689

$\Rightarrow$ Power of the test is $1 - 0.1151 = 0.8849$ $\square$