

- 38) An insurance company has a variable annuity linked to the S&P 500 index. A guaranteed minimum death benefit (GMDB) specifies the beneficiary will receive the greater of the account value and the original amount invested, if the policyholder dies within the first three years of the annuity contract. If the policyholder dies after three years, the beneficiary will receive the account value.

Out of every 1000 policies sold, the company expects 10 deaths in each of years one, two, and three. Thus they also expect that 970 will survive the first three years. Assume the deaths occur at the end of the year.

You are given the following at-the-money European call and put option prices, expressed as a percentage of the current value of the S&P 500 index.

Duration (years)	Call Price	Put Price
1	18.7%	15.8%
2	26.2%	20.6%
3	31.6%	23.4%

Calculate the expected value of the guarantee when the annuity is sold, expressed as a percentage of the original amount invested.

- (A) 0.23%
- (B) 0.32%
- (C) 0.52%
- (D) 0.60%
- (E) 0.76%

- 37) A policyholder owns a variable annuity contract with death benefit features defined as follows:
- (a) Guaranteed minimum death benefit (GMDB) with return of premium: the greater of the account value and the initial investment will be paid when the policyholder dies.
 - (b) Enhanced-income death benefit guarantee: 20% of the account value in excess of the initial investment amount will be added if the account value is greater than the initial investment when the policyholder dies.

Let T be the random variable denoting the future lifetime of the policyholder.

Let K be the initial investment amount of the variable annuity contract.

Let S_t be the value of the policyholder's account at time t .

You are given:

- T follows a distribution with probability density function $f(t), t > 0$.
- Given $T = t$, $p(t)$ is the payoff of a European put option based on account value S_t with strike price K .
- Given $T = t$, $c(t)$ is the payoff of a European call option based on account value S_t with strike price K .

Determine which one of the following statements is true.

- (A) The total death benefit payout for death at time t can be expressed as $\max(S_t, K) + 0.2 \times \max(K - S_t, 0)$.
- (B) The total death benefit payout for death at time t can be expressed as $\max(S_t - K, 0) + 0.2 \times \int_0^\infty p(t)f(t)dt$.
- (C) The expected value of the death benefit can be expressed as $K + \int_0^\infty c(t)f(t)dt + 0.2 \times \int_0^\infty p(t)f(t)dt$.
- (D) The expected value of the death benefit can be expressed as $K + \int_0^\infty p(t)f(t)dt + 0.2 \times \int_0^\infty c(t)f(t)dt$.
- (E) The expected value of the death benefit can be expressed as $K + 1.2 \times \int_0^\infty c(t)f(t)dt$.

- 36) Determine which one of the following statements regarding guarantees on variable annuity products is FALSE:
- (A) A guaranteed minimum death benefit (GMDB) with a return of premium guarantee is similar to a European put option with expiration contingent on the death of the policyholder or annuitant.
 - (B) A guaranteed minimum accumulation benefit (GMAB) with a return of premium guarantee is similar to a European put option with payment contingent on the policyholder surviving to the guarantee expiration date and the policy still being in force at that time.
 - (C) A guaranteed minimum withdrawal benefit (GMWB) provides a guarantee that the account value will not be less than the guaranteed withdrawal benefit base at any future time.
 - (D) A guaranteed minimum income benefit (GMIB) provides a guarantee on the future purchase rate for a traditional annuity.
 - (E) An earnings-enhanced death benefit is an optional benefit available with some variable annuity products that acts as a European call option with strike price equal to the original amount invested.

39) A variable annuity has the following guarantees:

- Guaranteed minimum death benefit with a return of premium guarantee.
- Guaranteed minimum accumulation benefit with a return of premium guarantee, effective 10 years from the date the policy is sold.
- Earnings-enhanced death benefit that pays the beneficiary an additional benefit equal to 20% of any increase in the account value.

The following notation is used:

- $P(T)$ denotes the value of a European put option on the annuity value, with the strike price equal to the original amount invested and time to expiration T .
- $C(T)$ denotes the value of a European call option on the annuity value, with the strike price equal to the original amount invested and time to expiration T .
- T_x denotes the future lifetime of the policyholder, and $f_{T_x}(t)$ denotes the probability density function of T_x .

Assuming no lapses, which expression below represents the combined value of all guarantees?

(A) $\int_0^{\infty} C(t)f_{T_x}(t)dt + \Pr(T_x \geq 10) \times P(10) + 0.2 \times \int_0^{\infty} C(t)f_{T_x}(t)dt$

(B) $\int_0^{\infty} C(t)f_{T_x}(t)dt + \Pr(T_x \geq 10) \times P(10) + 0.2 \times \int_0^{\infty} P(t)f_{T_x}(t)dt$

(C) $\int_0^{\infty} P(t)f_{T_x}(t)dt + \Pr(T_x \geq 10) \times P(10) + 0.2 \times \int_0^{\infty} C(t)f_{T_x}(t)dt$

(D) $\int_0^{\infty} P(t)f_{T_x}(t)dt + \Pr(T_x \leq 10) \times P(10) + 0.2 \times \int_0^{\infty} C(t)f_{T_x}(t)dt$

(E) $\int_0^{\infty} P(t)f_{T_x}(t)dt + \Pr(T_x \leq 10) \times P(10) + 0.2 \times \int_0^{\infty} P(t)f_{T_x}(t)dt$

- 40) Several lookback options are written on the same underlying index. They all expire in 3 years.

Let S_t denote the value at time t of the index on which the option is written.

The initial index price, S_0 , is 150.

The index price when the option expires, S_3 , is 200.

The maximum index price over the 3-year period is 210.

The minimum index price over the 3-year period is 120.

Calculate the sum of the payoffs for the following three lookback options:

- Standard lookback call
- Extrema lookback call with a strike price of 100
- Extrema lookback put with a strike price of 100

(A) 180

(B) 190

(C) 200

(D) 210

(E) 220