M339W: November 10th, 2021.

CAPM [cont'd].

Beta of a portfolio.

Let P be a portfolio such that

$$R_P = w_1 \cdot R_1 + w_2 \cdot R_2 + \dots + w_n \cdot R_n$$

$$\beta_P = \frac{\sigma_P}{\sigma_{Mkt}} \cdot \rho_{P,Mkt} = \frac{Cov[R_P, R_{Mkt}]}{Var[R_{Mkt}]} =$$

$$= \frac{Cov[w_1 \cdot R_1 + w_2 R_2 + \dots + w_n \cdot R_n, R_{Mkt}]}{Var[R_{Mkt}]}$$

$$= \sum_{i=1}^{n} w_i \underbrace{\frac{Cov[R_i, R_{Mkt}]}{Var[R_{Mkt}]}}_{= \beta_i} = \sum_{i=1}^{n} w_i \beta_i$$

7) Consider a portfolio of four stocks as displayed in the following table:

Stock	Weight	Beta
1	0.1	1.3
2	0.2	-0.6
3	0.3	β_3
4	0.4	1.1

✓

$\mathbb{E}[R_P] = 0.12$

Assume the expected return of the portfolio is 0.12, the annual effective risk-free rate is 0.05, and the market risk premium is 0.08.

$r_f = 0.05$

$\boxed{\mathbb{E}[R_{Mkt}] - r_f = 0.08}$

Assuming the Capital Asset Pricing Model holds, calculate β_3.

A) 0.80

B) 1.06

C) 1.42

D) 1.83

E) 2.17

$\beta_P = w_1 \cdot \beta_1 + w_2 \beta_2 + w_3 \beta_3 + w_4 \beta_4$

$\mathbb{E}[R_P] = r_P = r_f + \beta_P(\mathbb{E}[R_{Mkt}] - r_f)$

$0.12 = 0.05 + \beta_P(0.08)$

$\Rightarrow \beta_P = \dfrac{0.12 - 0.05}{0.08} = 0.875$

$0.875 = 0.1(1.3) + 0.2(-0.6) + 0.3\beta_3 + 0.4(1.1)$

$\beta_3 = 1.4167$

Task: Compare to the "official" sol'n.

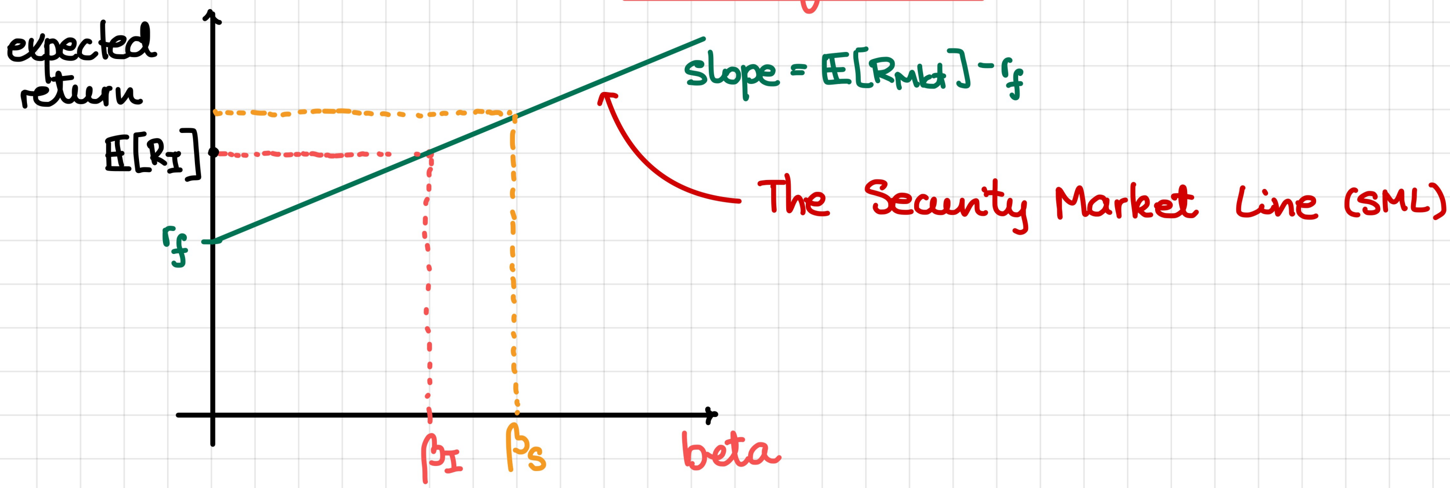
The Equity Cost of Capital.

In CAPM: for all investments I:

independent of investment I

$$\mathbb{E}[R_I] = r_I = \underbrace{r_f}_{\text{intercept}} + \beta_I \underbrace{\left(\mathbb{E}[R_{Mkt}] - r_f\right)}_{\text{the slope.}}$$

"independent argument"



Beta estimation.

Linear Regression.

Explanatory Random Variable: X

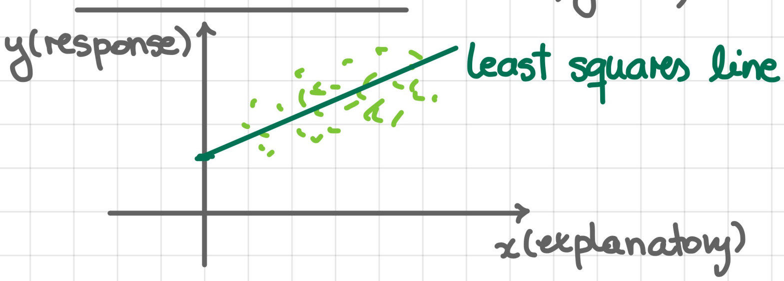
Response Random Variable: Y

Model: $\boxed{Y = \alpha + \beta \cdot X + \varepsilon}$ w/ $\varepsilon \sim \text{Normal}(0, \text{variance})$

intercept (α) slope (β)

variance: assume to be the same for all values of X

Observed values: (x_i, y_i), $i = 1..n$



$$\hat{\beta} = \frac{SD[Y]}{SD[X]} \cdot corr[X, Y]$$

$$Y = \alpha + \beta X + \varepsilon \qquad \text{(SLR)}$$

"Attacking" the SLR w/ the expectation.

$$\mathbb{E}[Y] = \alpha + \beta\,\mathbb{E}[X] + 0$$

They can be estimated from the least·squares line.

<u>In our applications:</u>

$$\underbrace{R_I - r_f}_{\substack{\text{excess return}\\ \text{for investment } I\\ \text{RESPONSE}}} = \alpha_I + \beta_I\,(\underbrace{R_{Mkt} - r_f}_{\substack{\text{excess return}\\ \text{of market}\\ \text{EXPLANATORY}}}) + \varepsilon_I$$

the error term

the intercept of the linear regression

the slope of the linear regression

Now, we see how we can estimate α_I and β_I from the observed values of excess returns across different time intervals.

Taking the expectation above, we get

$$\mathbb{E}[R_I] - r_f = \alpha_I + \beta_I\,(\mathbb{E}[R_{Mkt}] - r_f) + \overbrace{\mathbb{E}[\varepsilon_I]}^{=0}$$

$$\mathbb{E}[R_I] = \underbrace{r_f + \beta_I\,(\mathbb{E}[R_{Mkt}] - r_f)}_{\text{the Security Market Line}} + \alpha_I$$

the distance from the SML, i.e., the stock's <u>alpha</u>