

39) A variable annuity has the following guarantees:

- Guaranteed minimum death benefit with a return of premium guarantee.
- Guaranteed minimum accumulation benefit with a return of premium guarantee, effective 10 years from the date the policy is sold.
- Earnings-enhanced death benefit that pays the beneficiary an additional benefit equal to 20% of any increase in the account value.

The following notation is used:

- $P(T)$ denotes the value of a European put option on the annuity value, with the strike price equal to the original amount invested and time to expiration T .
- $C(T)$ denotes the value of a European call option on the annuity value, with the strike price equal to the original amount invested and time to expiration T .
- T_x denotes the future lifetime of the policyholder, and $f_{T_x}(t)$ denotes the probability density function of T_x .

Assuming no lapses, which expression below represents the combined value of all guarantees?

X (A) $\int_0^\infty C(t)f_{T_x}(t)dt + \Pr(T_x \geq 10) \times P(10) + 0.2 \times \int_0^\infty C(t)f_{T_x}(t)dt$

X (B) $\int_0^\infty C(t)f_{T_x}(t)dt + \Pr(T_x \geq 10) \times P(10) + 0.2 \times \int_0^\infty P(t)f_{T_x}(t)dt$

(C) $\int_0^\infty P(t)f_{T_x}(t)dt + \Pr(T_x \geq 10) \times P(10) + 0.2 \times \int_0^\infty C(t)f_{T_x}(t)dt$ 😊

X (D) $\int_0^\infty P(t)f_{T_x}(t)dt + \Pr(T_x \leq 10) \times P(10) + 0.2 \times \int_0^\infty C(t)f_{T_x}(t)dt$

X (E) $\int_0^\infty P(t)f_{T_x}(t)dt + \Pr(T_x \leq 10) \times P(10) + 0.2 \times \int_0^\infty P(t)f_{T_x}(t)dt$

→: GMDB $\max(S(T_x), K) = S(T_x) + \underbrace{\max(K - S(T_x), 0)}_{\text{Put}}$

More Exotic Options.

Shout Options.

The owner of the shout option has the right to select a date (**shout!**) during the life of the option. Then the price of the underlying asset is used to calculate the payoff on the exercise date provided that gives a higher payoff.

S^* ... the price of the underlying on the shout date
Payoff

Shout call: $\max(0, S(T) - K, S^* - K)$

Shout put: $\max(0, K - S(T), K - S^*)$

Purpose: This is a hedge against the policyholder resetting the pmt guarantee once.
If the reset takes place, then the payoff of the annuity is

$$\max(K, S(T_x), S^*) = K + \max(0, S(T_x) - K, S^* - K)$$

Chooser Options.

The owner of the option decides whether the option becomes a call or a put on the choice date.

Rainbow Options.

... when the payoff depends on two or more risky assets,

e.g., $\max(S(T) - K, A(T) - K, 0)$ is the payoff of a rainbow call

$\max(K - S(T), K - A(T), 0)$ is the payoff of a rainbow put

Forward Start Options.

...a right to buy an option @ a later time w/ the strike dependent on the asset price @ that time.