M339W : May 4th, 2022.

Options embedded in insurance products [cont'd].

II. Guaranteed minimum accumulation benefit (GMAB).

It guarantees the minimum value in the account @ a pre-specified time (m) should the annuitant live to that time.

K ... the guaranteed amount

The GMAB will pay: $\max(K - S(m), 0)$ (like the put-option payoff) if the policyholder is still alive.

=> Its total value is

$$P(m) \cdot \mathbb{P}[T_x \geq m]$$

w/ P(m) being the price of a put option w/ strike K and exercise date @ time-m,

III. Guaranteed minimum withdrawal benefit (GMWB).

GMWB guarantees that the policyholder can withdraw a specific pre-determined sum per year after they reach a certain age.

IV. Guaranteed minimum income benefit (GMIB).

The price of a whole-life annuity is guaranteed @ a particular age.

36) Determine which one of the following statements regarding guarantees on variable annuity products is FALSE:

(A) A guaranteed minimum death benefit (GMDB) with a return of premium guarantee is similar to a European put option with expiration contingent on the death of the policyholder or annuitant.

(B) A guaranteed minimum accumulation benefit (GMAB) with a return of premium guarantee is similar to a European put option with payment contingent on the policyholder surviving to the guarantee expiration date and the policy still being in force at that time.

(C) A guaranteed minimum withdrawal benefit (GMWB) provides a guarantee that the account value will not be less than the guaranteed withdrawal benefit base at any future time.

(D) A guaranteed minimum income benefit (GMIB) provides a guarantee on the future purchase rate for a traditional annuity.

(E) An earnings-enhanced death benefit is an optional benefit available with some variable annuity products that acts as a European call option with strike price equal to the original amount invested.

Inflation Indexing.

This is the property of some pensions (e.g., Social Security).

Denote by I_t the consumer price index (CPI) @ time t.

Denote by P_t the pension pmt @ time t.

Let the first pmt be @ time 0. That pmt equals P_0.

Then, for the pmt @ time 1, we have

$$P_1 = \max\left(P_0, P_0\left(\frac{I_1}{I_0}\right)\right)$$

$$\Rightarrow P_1 = P_0 + P_0 \boxed{\max\left(0, \left(\frac{I_1}{I_0}\right) - 1\right)}$$

$$P_1 = P_0\left(\frac{I_1}{I_0}\right) + P_0 \boxed{\max\left(1 - \left(\frac{I_1}{I_0}\right), 0\right)}$$

Recursively, we set

$$P_t = \max\left(P_{t-1}, P_0\left(\frac{I_t}{I_0}\right)\right)$$

$$\Rightarrow P_t = \max_{0 \le s \le t}\left(P_0\left(\frac{I_s}{I_0}\right)\right)$$

$$\Rightarrow P_t = P_0\left(\frac{I_t}{I_0}\right) + \left(\max_{0 \le s \le t}\left(P_0\left(\frac{I_s}{I_0}\right)\right) - P_0\left(\frac{I_t}{I_0}\right)\right)$$

like the standard lookback put

Lookback Options.

$S(t)$ ... the account value @ time t

T ... exercise date

$$m(T) = \min_{0 \le s \le T} S(s)$$

$$M(T) = \max_{0 \le s \le T} S(s)$$

	Payoff
• Standard lookback call:	$S(T) - m(T)$
• Standard lookback put:	$M(T) - S(T)$

Say, K denotes the strike price.

- Extrema lookback call: $\left(M(T)-K\right)_+$
- Extrema lookback put: $\left(K-m(T)\right)_+$

40) Several lookback options are written on the same underlying index. They all expire in 3 years.

Let S_t denote the value at time t of the index on which the option is written.

The initial index price, S_0, is 150.

The index price when the option expires, S_3, is 200.

The maximum index price over the 3-year period is 210.

The minimum index price over the 3-year period is 120.

Calculate the sum of the payoffs for the following three lookback options:

- Standard lookback call $S(T) - m(T) = 200 - 120 = 80$
- Extrema lookback call with a strike price of 100 $(M(T) - K)_+ = (210 - 100)_+ = 110$
- Extrema lookback put with a strike price of 100 $(K - m(T))_+ = (100 - 120)_+ = 0$

answer: $80 + 110 = 190$

(A) 180

(B) 190

(C) 200

(D) 210

(E) 220