M339W: May 2nd, 2022.

Read the Study Note!

Options Embedded in Insurance Products.

variable annuties.

I. Guaranteed Minimum Death Benefit (GMDB).

K... guaranteed minimum amount paid @ death;
If K is equal to the original premium, this is called the return of premium guarantee

$S(T)$... account value @ time T

If the policyholder dies @ time T_x, the total amount paid is:

$$\boxed{\max(S(T_x), K)}$$

Note:

$$\max(S(T_x), K) = S(T_x) + \boxed{\max(0, K - S(T_x))}$$

like the payoff of a put

For a vanilla put: $v_p(S(T), T) = (K - S(T))_+$

Let f_{T_x} be the pdf of the time·of·death T_x of x·year·olds.

$$\boxed{\int_0^{+\infty} P(t) f_{T_x}(x)\,dt}$$

w/ $P(t)$... the price of the put w/ strike K and exercise date t.

38) An insurance company has a variable annuity linked to the S&P 500 index. A guaranteed minimum death benefit (GMDB) specifies the beneficiary will receive the greater of the account value and the original amount invested, if the policyholder dies within the first three years of the annuity contract. If the policyholder dies after three years, the beneficiary will receive the account value.

Out of every 1000 policies sold, the company expects 10 deaths in each of years one, two, and three. Thus they also expect that 970 will survive the first three years. Assume the deaths occur at the end of the year.

You are given the following at-the-money European call and put option prices, expressed as a percentage of the current value of the S&P 500 index.

Duration (years)	Call Price	Put Price
1	18.7%	15.8%
2	26.2%	20.6%
3	31.6%	23.4%

Calculate the expected value of the guarantee when the annuity is sold, expressed as a percentage of the original amount invested.

(A) 0.23%

(B) 0.32%

(C) 0.52%

(D) 0.60%

(E) 0.76%

The deaths are assumed to be discrete.
So, our integral "becomes" a sum:

$0.01\cdot(0.158) + 0.01(0.206) + 0.01(0.234)$

$= \underline{0.00598}$ $\square$

In addition, there can be an earnings-enhanced death benefit. This benefit is a percentage of the excess above the guarantee of the account value (if any).

The payoff of this guarantee will be:

$$\boxed{p\% \cdot \boxed{\max\left(S(T_x) - K, 0\right)}}$$

like the payoff of a call option

The total value of the benefit can be expressed as:

$$\boxed{p\% \int_0^{+\infty} C(t) f_{T_x}(t)\, dt}$$

w/ $C(t)$ being the price of a call w/ strike K and exercise date t

37) A policyholder owns a variable annuity contract with death benefit features defined as follows:

(a) Guaranteed minimum death benefit (GMDB) with return of premium: the greater of the account value and the initial investment will be paid when the policyholder dies.

(b) Enhanced-income death benefit guarantee: 20% of the account value in excess of the initial investment amount will be added if the account value is greater than the initial investment when the policyholder dies.

Let T be the random variable denoting the future lifetime of the policyholder. $T_x = T$

Let K be the initial investment amount of the variable annuity contract.

Let S_t be the value of the policyholder's account at time t.

You are given:

- T follows a distribution with probability density function $f(t), t > 0$.
- Given $T = t$, $p(t)$ is the payoff of a European put option based on account value S_t with strike price K.
- Given $T = t$, $c(t)$ is the payoff of a European call option based on account value S_t with strike price K.

Determine which one of the following statements is true.

(A) The total death benefit payout for death at time t can be expressed as $\max(S_t, K) + 0.2 \times \max(K - S_t, 0)$.

(B) The total death benefit payout for death at time t can be expressed as $K+$ $\max(S_t - K, 0) + 0.2 \times \int_0^\infty p(t) f(t)\,dt$.

(C) The expected value of the death benefit can be expressed as $K + \int_0^\infty c(t) f(t)\,dt + 0.2 \times \int_0^\infty p(t) f(t)\,dt$.

(D) The expected value of the death benefit can be expressed as $K + \int_0^\infty p(t) f(t)\,dt + 0.2 \times \int_0^\infty c(t) f(t)\,dt$.

(E) The expected value of the death benefit can be expressed as $K + 1.2 \times \int_0^\infty c(t) f(t)\,dt$.

The intended answer!

→: The expected value of the death benefit = ?

$$\underbrace{\max(S_t, K)}_{\text{GMDB}} + \underbrace{0.2 \max(S_t - K, 0)}_{\text{EEDB}} =$$

$$= K + \max(S_t - K, 0) + 0.2 \max(S_t - K, 0)$$

$$= K + 1.2 \boxed{\max(S_t - K, 0)}$$

like the call payoff : $c(t)$

=> the expected payoff is

$$K + 1.2\, \mathbb{E}\left[\int_0^{+\infty} c(t) f(t)\, dt \right]$$