M339W: March 30th, 2022.

50. Assume the Black-Scholes framework.

You are given the following information for a stock that pays dividends continuously at a rate proportional to its price.

(i) The current stock price is 0.25.

(ii) The stock's volatility is 0.35.

(iii) The continuously compounded expected rate of stock-price appreciation is 15%.

Calculate the upper limit of the 90% lognormal confidence interval for the price of the stock in 6 months.

(A) 0.393

(B) 0.425

(C) 0.451

(D) 0.486

(E) 0.529

51-53. DELETED

54. Assume the Black-Scholes framework. Consider two nondividend-paying stocks whose time-t prices are denoted by $S_1(t)$ and $S_2(t)$, respectively.

You are given:

(i) $S_1(0) = 10$ and $S_2(0) = 20$.

(ii) Stock 1's volatility is 0.18. $\boxed{\sigma_1 = 0.18}$

(iii) Stock 2's volatility is 0.25. $\boxed{\sigma_2 = 0.25}$

(iv) The correlation between the continuously compounded returns of the two stocks is –0.40. $\boxed{\rho = -0.40}$

(v) The continuously compounded risk-free interest rate is 5%. $\boxed{r = 0.05}$

$V_{SO}(0) = 1.632 \longrightarrow$ (vi) A one-year European option with payoff $\max\{\underbrace{\min[2S_1(1), S_2(1)]}_{=:Y(1)} - 17, 0\}$ has a current (time-0) price of 1.632. SO

← SPECIAL PUT

Consider a European option that gives its holder $\boxed{\text{the right to sell}}$ either two shares of Stock 1 or one share of Stock 2 at a $\boxed{\text{price of 17}}$ one year from now.

T = 1

Calculate the current (time-0) price of this option.

$V_{SP}(0) = ?$

Focus on the "special put":

Its payoff will be

$\left(17 - \boxed{\min(2\cdot S_1(1), S_2(1))}\right)_+$

$=: Y(1)$

(A) 0.67

(B) 1.12

(C) 1.49

(D) 5.18

(E) 7.86

Looks like the payoff of a 17-strike put w/ underlying Y and exercise date 1.

55. Assume the Black-Scholes framework. Consider a 9-month at-the-money European put option on a futures contract. You are given:

(i) The continuously compounded risk-free interest rate is 10%.

(ii) The strike price of the option is 20.

(iii) The price of the put option is 1.625.

If three months later the futures price is 17.7, what is the price of the put option at that time?

(A) 2.09

(B) 2.25

(C) 2.45

(D) 2.66

(E) 2.83

56-76. DELETED

Put-Call Parity.

$$\underbrace{V_{SO}(0)}_{1.632} - \boxed{V_{SP}(0)} = \boxed{F^P_{0,T}(Y)} - \underbrace{PV_{0,T}(K)}_{=17\cdot e^{-0.05}}$$

?

The price which should be paid @ time 0 to receive $\boxed{Y(1) = \min(2S_1(1), S_2(1))}$ @ time 1.

Focus on Y:

$$Y(1) = \min(2S_1(1), S_2(1))$$

$$Y(1) = S_2(1) + \min(2S_1(1) - S_2(1), 0)$$

$$Y(1) = \boxed{S_2(1)} - \boxed{\max(S_2(1) - 2S_1(1), 0)}$$

$\min(a,b) = -\max(-a,-b)$

Generally: The prepaid forward
No dividends: Outright Purchase

Time·0: $F^P_{0,1}(S_2) = S_2(0) = 20$

Exchange call
w/ underlying S_2
and strike asset $2 \cdot S_1$

$$2 \cdot S_1(T) = 2 \cdot S_1(0) e^{(r - \delta_1 - \frac{\sigma_1^2}{2}) \cdot T + \sigma_1 \sqrt{T} \cdot Z_1}$$

w/ $Z_1 \sim N(0,1)$

$\Downarrow$

$2S_1$ has the same δ_1 and σ_1 as the original S_1.

$V_{EC}(0, S_2, 2S_1) = ?$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$$
$$= (0.18)^2 + (0.25)^2 - 2(-0.4)(0.18)(0.25)$$
$$= 0.1309$$

$$\sigma = \underline{0.3618}$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S_2(0)}{2S_1(0)}\right) + \frac{1}{2}\sigma^2 \cdot T\right] = \frac{1}{2}\sigma\sqrt{T}$$

$\ln\left(\frac{S_2(0)}{2S_1(0)}\right) = 0$

"at·the·money"

$$d_2 = d_1 - \sigma\sqrt{T} = \frac{1}{2}\sigma\sqrt{T} - \sigma\sqrt{T} = -\frac{1}{2}\sigma\sqrt{T}$$

$$V_{EC}(0, S_2, 2S_1) = S_2(0) \cdot N(d_1) - 2S_1(0) N(d_2)$$

$$= 20\left(N\left(\tfrac{1}{2}\sigma\sqrt{T}\right) - N\left(-\tfrac{1}{2}\sigma\sqrt{T}\right)\right)$$

$$= 20\left(2N\left(\tfrac{1}{2}\sigma\sqrt{T}\right) - 1\right)$$

$$= 20\left(2 \cdot N(0.1809) - 1\right) = 2.871079$$

$\bigstar \Rightarrow \quad 1.632 - V_{sp}(0) = (20 - 2.871079) - 17e^{-0.05}$

$$V_{sp}(0) = 1.632 - (20 - 2.871079) + 17e^{-0.05}$$

$$= \boxed{0.6739792}$$