M339W : March 23rd, 2022.

Delta Hedging.

Market makers.

- immediacy
- (inventory)

$\Rightarrow$ exposure to risk $\Rightarrow$ hedge

Say, a market maker writes an option whose value f'tion is:

$$\boxed{v(s,t)} \quad ✓$$

At time 0, they write the option. So, the get $v(S(0),0)$.

At time t, the value of their position is

$$\boxed{-v(s,t)} \quad ✓$$

To (partially) hedge their exposure to risk, they construct a portfolio which has zero sensitivity to small perturbations in the stock price, i.e., they aim to create a

delta-neutral portfolio,

i.e.,

a portfolio for which $\boxed{\Delta_{Port}(s,t)=0}$

Theoretically, one would continuously rebalance the portfolio to maintain the zero delta; this is possible w/ no transaction costs.

Practically, continuous rebalancing is impossible and there are transaction costs.

At time 0, we want to trade so that

$$\boxed{\Delta_{Port}(S(0),0)=0}$$

The most straightforward strategy is to trade in the underlying asset itself.

At time t, let $\boxed{N(s,t)}$ denote the necessary number of shares of stock so that the total portfolio is Δ-neutral.

The total value of the portfolio is:

$$\frac{\partial}{\partial s}\Big| \quad v_{Port}(s,t) = -v(s,t) + \boxed{N(s,t)}\cdot s$$

$$\Delta_{Port}(s,t) = -\Delta(s,t) + N(s,t) = 0$$

Δ-neutral

$$\Rightarrow \quad \boxed{N(s,t) = \Delta(s,t)}$$

Example. A market maker writes a call @ time·0.

At any time·t, the market maker's position is:

$$-v_c(s,t)$$

$\Rightarrow$ They have to maintain $N(s,t) = \Delta_c(s,t)$ in the Δ-hedge.

In particular, @ time·0:

$$\underline{N(S(0),0) = e^{-\delta\cdot T}\cdot N(d_1(S(0),0))} > 0$$

i.e., the writer of the option must long this much of a share.

Example. A market maker writes a put @ time·0.

At time·t, their position is: $-v_p(s,t)$

$\Rightarrow$ They have to maintain $N(s,t) = \Delta_p(s,t)$ in their Δ-hedge.

In particular, @ time·0:

$$\underline{N(S(0),0) = -e^{-\delta T}\cdot N(-d_1(S(0),0))} < 0,$$

i.e., the market maker should ___short___ this much of a share.

46. You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is 6 months.

(ii) $u/d = 4/3$, where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down.

(iii) The risk-neutral probability of an up move is 1/3.

(iv) The initial futures price is 80.

(v) The continuously compounded risk-free interest rate is 5%.

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

Determine $C_{II} - C_I$.

(A) 0

(B) 0.022

(C) 0.044

(D) 0.066

(E) 0.088

47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.

You are given the following information:

(i) The risk-free interest rate is constant.

(ii)

	when the option was written Several months ago @ time 0	when the positions are closed out Now time t
Stock price	$\boxed{\$40.00}$	$\boxed{\$50.00}$
Call option price	$\boxed{\$\ 8.88}$	$\boxed{\$14.42}$
Put option price	\$ 1.63	\$ 0.26
Call option delta	$\boxed{0.794}$	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.

Calculate her profit.

$\boxed{\text{Profit}} = \text{Payoff} - FV(\text{Initial Cost})$

(A) $11

(B) $24

(C) $126

(D) $217

(E) $240

0 — options written

t — "several months" — This is when we calculate the profit.

T = 1 — exercise

$\boxed{\text{Profit (@ time } t) = \text{Wealth (@ time } t) - FV_{0,t}(\text{Init. Cost})}$

48. DELETED

49. You use the usual method in McDonald and the following information to construct a one-period binomial tree for modeling the price movements of a nondividend-paying stock. (The tree is sometimes called a forward tree).

(i) The period is 3 months.

(ii) The initial stock price is $100.

(iii) The stock's volatility is 30%.

(iv) The continuously compounded risk-free interest rate is 4%.

At the beginning of the period, an investor owns an American put option on the stock. The option expires at the end of the period.

Determine the smallest integer-valued strike price for which an investor will exercise the put option at the beginning of the period.

(A) 114

(B) 115

(C) 116

(D) 117

(E) 118

- Initial Cost: $-100\, v_c(S(0),0) + 100 \cdot \Delta_c(S(0),0) \cdot S(0)$

$$= 100\left(-8.88 + 0.794 \cdot 40\right) = 2{,}288. \checkmark$$

- Wealth @ time t: $-100\, v_c(S(t),t) + 100 \cdot \boxed{\Delta_c(S(0),0)} \cdot S(t) =$

$$= 100\left(-14.42 + 0.794 \cdot 50\right) = 2{,}528.$$

- Profit (@ time t) $= 2{,}528 - e^{rt}\, 2{,}288$

$e^{rt} = \text{?}$

Idea: Put-Call Parity

no div

$= S(0)$

At time 0: $v_c(S(0),0) - v_p(S(0),0) = F^P_{0,T}(S) - Ke^{-rT}$

$$8.88 - 1.63 = 40 - \boxed{Ke^{-rT}}$$

(0) $Ke^{-rT} = 40 - 7.25 = 32.75$

no div

$= S(t)$

At time t: $v_c(S(t),t) - v_p(S(t),t) = F^P_{t,T}(S) - Ke^{-r(T-t)}$

$$14.42 - 0.26 = 50 - \boxed{Ke^{-r(T-t)}}$$

(t) $Ke^{-r(T-t)} = 50 - 14.16 = 35.84$

$$\frac{(t)}{(0)} = \frac{\cancel{Ke^{-rT}}}{\cancel{Ke^{-r(T-t)}}} = \frac{35.84}{32.75}$$

$$e^{rt} = 1.09435$$

Profit (@ time t) $= 2{,}528 - 1.09435 \cdot 2{,}288 = 24.1272$ $\square$