Problem 3.5. The current price of a continuous-dividend-paying stock is $100 per share. Its volatility is given to be 0.2 and its dividend yield is 0.03.

The continuously compounded risk-free interest rate equals $\boxed{0.06.}$

Consider a $95-strike European put option on the above stock with nine months to expiration. Using a three-period forward binomial tree, find the price of this put option.

$n=3$ $\boxed{T = 3/4}$

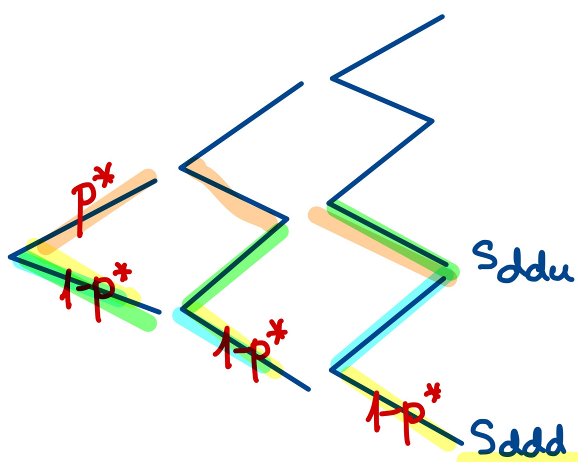
(a) $2.97
(b) $3.06
(c) $3.59
(d) $3.70
(e) None of the above.

$$h = \frac{T}{n} = \frac{1}{4}$$

$\rightarrow$:

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.2\sqrt{1/4}}} = \frac{1}{1+e^{0.1}} = \underline{0.4750}$$

$$\begin{cases} u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(0.06-0.03)(0.25)+0.1} = e^{0.1075} \quad (= \underline{1.1135}) \\ d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{0.0075-0.1} = e^{-0.0925} \quad (= \underline{0.9116}) \end{cases}$$



$V_{uuu} = 0$

$V_{duu} = 0$

$V_{ddu} = (K - S_{ddu})_+$ w/ $3(1-p^*)^2 \cdot p^*$

$V_{ddd} = (K - S_{ddd})_+$ w/ $(1-p^*)^3$

n periods $\Rightarrow$ $n+1$ possible final stock prices

$$S_{ddd} = S(0)\, d^3 = (100) \cdot (e^{-0.0925})^3 \approx 75.7676 < 95 = K \text{ i.t.m.}$$

$$S_{ddu} = S(0)\, d^2 \cdot u = (100)(e^{-0.0925})^2(e^{0.1075})$$
$$= 92.5427 < 95 \text{ itm}$$

$$S_{duu} = S(0)\, d \cdot u^2 = (100)\, e^{-0.0925}(e^{0.1075})^2 =$$
$$= 113.0319 > 95 \text{ o.o.m.}$$

$$V_p(0) = e^{-rT}\,\mathbb{E}^*[V_p(T)]$$

$$= e^{-0.06(0.75)}\Big[(95 - S_{ddd})_+ \cdot (1-p^*)^3$$

$$+ (95 - S_{ddu})_+ \cdot 3(1-p^*)^2 \cdot p^*\Big]$$

$$= 3.5818 \quad \blacksquare$$

$\delta = 0$

Problem 3.6. Consider a non-dividend-paying stock whose current price is $100 per share. Its volatility is given to be 0.25. You model the evolution of the stock price over the following year using a two-period forward binomial tree.

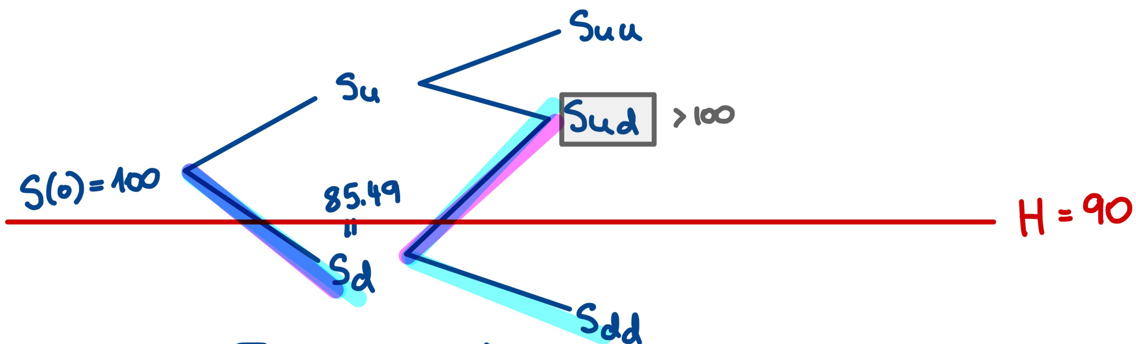
The continuously compounded risk-free interest rate is 0.04.

Consider a $110-strike, one-year **down-and-in** put option with a barrier of $90 on the above stock. What is the price of this option consistent with the above stock-price model?

(a) About $10.23
(b) About $11.55
(c) About $11.78
(d) About $11.90
(e) None of the above.

$\rightarrow$:

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.25\sqrt{0.5}}} \approx 0.4559$$



$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{0.04(0.5)+0.25\sqrt{0.5}} = 1.2175$$

$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{0.04(0.5)-0.25\sqrt{0.5}} = 0.8549$$

$S_{dd} = S(0)d^2 = 73.0854 \Rightarrow V_{dd} = (110 - S_{dd})_+ = 36.9146$ w/ probab. $(1-p^*)^2$

$S_{du} = S(0)du = 104.08 \Rightarrow V_{du} = (110 - S_{du})_+ = 5.92$ $p^*(1-p^*)$

$$V(0) = e^{-0.04(1)}\left[p^*(1-p^*)\cdot V_{du} + (1-p^*)^2 \cdot V_{dd}\right] = 11.91$$