M339W: January 27th, 2021.

Currency Options.

- Domestic Currency (DC) ...

  r_D = continuously compounded, risk-free interest rate
- Foreign Currency (FC) ...

  r_F = ccrfir

$x(t)$, $t \geq 0$ ... EXCHANGE RATE from FC to DC

(in words, 1 unit of FC is worth $x(t)$ units of DC @ time t)

Recall:

- $F^P_{0,T}(x) = e^{-r_F \cdot T} \cdot x(0)$ in DC
- $F_{0,T}(x) = e^{(r_D - r_F) \cdot T} \cdot x(0)$
- Put-Call Parity

strike in DC

$$V_C(0) - V_P(0) = F^P_{0,T}(x) - PV_{0,T}(K)$$
$$= x(0)e^{-r_F \cdot T} - Ke^{-r_D \cdot T}$$

Analogy: Foreign currency $\longleftrightarrow$ Continuos dividend stocks

$x(t) \longleftrightarrow S(t)$

$r_F \longleftrightarrow \delta$

The same analogy holds for the binomial option pricing. We will see this via pricing by replication.

Binomial model for the exchange rate

$$
\begin{tikzcd}[column sep=large]
 & x_u = u \cdot x(0) \\
x(0) \arrow[ur] \arrow[dr] & \\
 & x_d = d \cdot x(0)
\end{tikzcd}
\qquad
\begin{aligned}
V_u &= v(x_u) = \Delta e^{r_F \cdot h} \cdot x_u + B e^{r_D \cdot h} \\
V_d &= v(x_d) = \Delta e^{r_F \cdot h} \cdot x_d + B e^{r_D \cdot h}
\end{aligned}
$$

$$\underbrace{\qquad\qquad}_{h = T}$$

Completely analogous to the stock options!

Let $v(\cdot)$ be the payoff of the European option on the foreign currency.

The Replicating Portfolio:

☆
- Δ ... # of units of the FC invested in @ time 0 and then "earning" r_F for the duration of our binomial period
- B ... our risk-free investment in the DC (@ the risk-free interest rate r_D)

$\Rightarrow$ We mostly use the risk-neutral pricing formula w/ the risk-neutral probability

$$p^* := \frac{e^{(r_D - r_F) \cdot h} - d}{u - d}$$

4. For a two-period binomial model, you are given:

(i) Each period is one year.

(ii) The current price for a nondividend-paying stock is 20.

(iii) $u = 1.2840$, where u is one plus the rate of capital gain on the stock per period if the stock price goes up.

(iv) $d = 0.8607$, where d is one plus the rate of capital loss on the stock per period if the stock price goes down.

(v) The continuously compounded risk-free interest rate is 5%.

Calculate the price of an American call option on the stock with a strike price of 22.

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

$T = \frac{3}{4}$ $\quad$ \$... DC $\quad$ £... FC

5. Consider a 9-month dollar-denominated American put option on British pounds. You are given that:

(i) The current exchange rate is 1.43 US dollars per pound. $x(0) = 1.43$ (\$/£)

(ii) The strike price of the put is 1.56 US dollars per pound. $K = 1.56$

(iii) The volatility of the exchange rate is $\sigma = 0.3$.

(iv) The US dollar continuously compounded risk-free interest rate is 8%. $r_{\$} = 0.08$

(v) The British pound continuously compounded risk-free interest rate is 9%. $\boxed{r_{£} = 0.09}$

$n = 3$

Using a three-period binomial model, calculate the price of the put.

The length of each period:

$h = 1/4$

(A) 0.23

(B) 0.25

(C) 0.27

(D) 0.29

(E) 0.31

This is a <u>forward tree</u>:

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.3\sqrt{1/4}}} = \frac{1}{1+e^{0.15}} \cong 0.4626$$

$$u = e^{(r_D - r_F)\cdot h + \sigma\sqrt{h}} = e^{(0.08-0.09)(0.25)+0.15} = e^{0.1475} = 1.1589$$

$$d = e^{(r_D - r_F)\cdot h - \sigma\sqrt{h}} = e^{(0.08-0.09)(0.25)-0.15} = e^{-0.1525} = 0.8586$$

Complete problem @ home!
Optional, but recommended, extra credit HW!