33. You own one share of a nondividend-paying stock. Because you worry that its price may drop over the next year, you decide to employ a *rolling insurance strategy*, which entails obtaining one 3-month European put option on the stock every three months, with the first one being bought immediately.

You are given:

(i) The continuously compounded risk-free interest rate is 8%.

(ii) The stock's volatility is 30%.

(iii) The current stock price is 45.

(iv) The strike price for each option is 90% of the then-current stock price.

Your broker will sell you the four options but will charge you for their total cost now.

Under the Black-Scholes framework, how much do you now pay your broker?

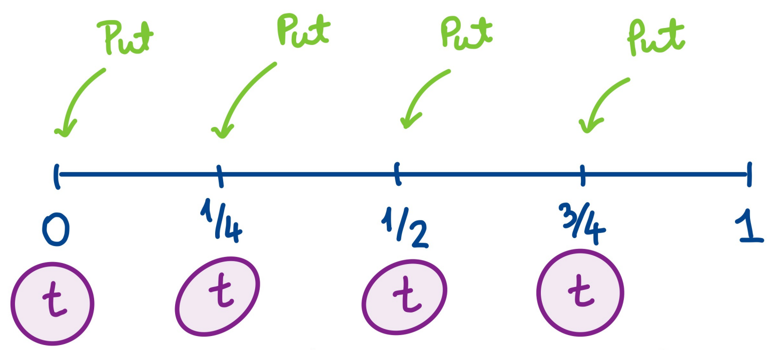
(A) 1.59

(B) 2.24

(C) 2.86

(D) .48

(E) 3.61



t... represents the valuation dates

34-39. DELETED

For each of the four puts in the rolling insurance strategy:

- one quarter-year to exercise
- $K_t = 0.9 \cdot S(t)$

For every t @ which a put option is received:

$$d_1(t) = \frac{1}{\sigma\sqrt{1/4}}\left[\ln\left(\frac{\cancel{S(t)}}{0.9\cdot\cancel{S(t)}}\right) + \left(r + \frac{\sigma^2}{2}\right)(1/4)\right]$$

$$d_1(t) = \frac{1}{0.3(0.5)}\left[-\ln(0.9) + \left(0.08 + \frac{0.09}{2}\right)(0.25)\right] = 0.9107$$

$$d_2(t) = 0.9107 - 0.3(0.5) = 0.7607$$

$\underline{N(-d_1(t))} = \text{pnorm}(-0.9107) = \underline{0.1812267}$

$\underline{N(-d_2(t))} = \text{pnorm}(-0.7607) = \underline{0.2234181}$

$$V_P(t) = 0.9\, S(t) \cdot e^{-0.08(0.25)} \cdot (0.2234) - S(t)\,(0.1812)$$

$$V_P(t) = S(t)\left(0.9e^{-0.02} \cdot 0.2234 - 0.1812\right) = S(t) \cdot 0.01588$$

$\Rightarrow$ Note that for every "put delivery" date $t = 0, 1/4, 1/2, 3/4$, the value of the put @ that time is equal to the value of a fraction 0.01588 of a share of the underlying stock.

$\Rightarrow$ Today, the worth of the put to be delivered @ time t is:

$$0.01588 \cdot F^P_{0,t}(S)$$

prepaid forward price for delivery @ time t

The stock pays no dividends $\Rightarrow F^P_{0,t}(S) = S(0) = 45$

Altogether,

$$\underbrace{4}_{\#\text{ of puts}} \cdot\, 0.01588 \cdot 45 = \underline{2.86}$$

is the price of the rolling insurance strategy.

Gap Options.

T ... exercise date

K_s ... strike price

K_t ... trigger price

$$V_{GC}(T) = (S(T) - K_s) \cdot \mathbb{I}_{[S(T) \geq K_t]}$$

$$V_{GP}(T) = (K_s - S(T)) \cdot \mathbb{I}_{[S(T) < K_t]}$$

Black-Scholes Pricing.

$$V_{GC}(0) = S(0)e^{-\delta T} \cdot N(d_1) - K_s \cdot e^{-rT} \cdot N(d_2)$$

$$V_{GP}(0) = K_s e^{-rT} \cdot N(-d_2) - S(0) e^{-\delta T} \cdot N(-d_1)$$

w/ $$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K_t}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right) \cdot T\right]$$

and $$d_2 = d_1 - \sigma\sqrt{T}$$

Example. Consider a gap call w/ a given trigger price K_t. Then, we can find a strike price K_s so that the price of the gap call is zero.

$\longrightarrow$:

$$0 = S(0)e^{-\delta T} \cdot N(d_1) - K_s \cdot e^{-rT} \cdot N(d_2)$$

$$\Rightarrow \quad K_s = \frac{S(0)e^{-\delta T} \cdot N(d_1)}{e^{-rT} \cdot N(d_2)} = S(0)e^{(r-\delta) \cdot T} \cdot \frac{N(d_1)}{N(d_2)}$$

$\blacksquare$