M339W: February 25th, 2022.

B.S Price of a European Call.

Review:

$$V_c(0) = S(0)e^{-\delta T}\cdot N(d_1) - Ke^{-rT}\cdot N(d_2)$$

w/

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r-\delta+\frac{\sigma^2}{2}\right)\cdot T\right]$$

and

$$d_2 = d_1 - \sigma\sqrt{T}$$

Black-Scholes Price of a European Put.

By put-call parity:

$$V_c(0) - V_P(0) = F^P_{0,T}(S) - PV_{0,T}(K)$$

$$V_P(0) = V_c(0) - F^P_{0,T}(S) + PV_{0,T}(K)$$

$$= S(0)e^{-\delta T}\cdot N(d_1) - Ke^{-rT}\cdot N(d_2)$$

$$- S(0)e^{-\delta T} \qquad + Ke^{-rT}$$

$$= S(0)e^{-\delta T}\big(\underbrace{N(d_1)-1}_{-N(-d_1)}\big) + Ke^{-rT}\big(\underbrace{-N(d_2)+1}_{N(-d_2)}\big)$$

symmetry of N(0,1):

$$\boxed{V_P(0) = Ke^{-rT}\cdot N(-d_2) - S(0)e^{-\delta T}\cdot N(-d_1)}$$

3. You are asked to determine the price of a European put option on a stock. Assuming the Black-Scholes framework holds, you are given:

(i) The stock price is $100. $S(0)=100$

(ii) The put option will expire in 6 months. $T=1/2$

(iii) The strike price is $98. $K=98$

(iv) The continuously compounded risk-free interest rate is $r = 0.055$.

(v) $\delta = 0.01$

(vi) $\sigma = 0.50$

Calculate the price of this put option.

(A) $3.50

(B) $8.60

(C) $11.90

(D) $16.00

(E) $20.40

$$d_1 = \frac{1}{0.5\sqrt{0.5}}\left[\ln\left(\frac{100}{98}\right) + \left(0.055 - 0.01 + \frac{0.25}{2}\right)(0.5)\right]$$

$$d_1 = \underline{0.2976}$$

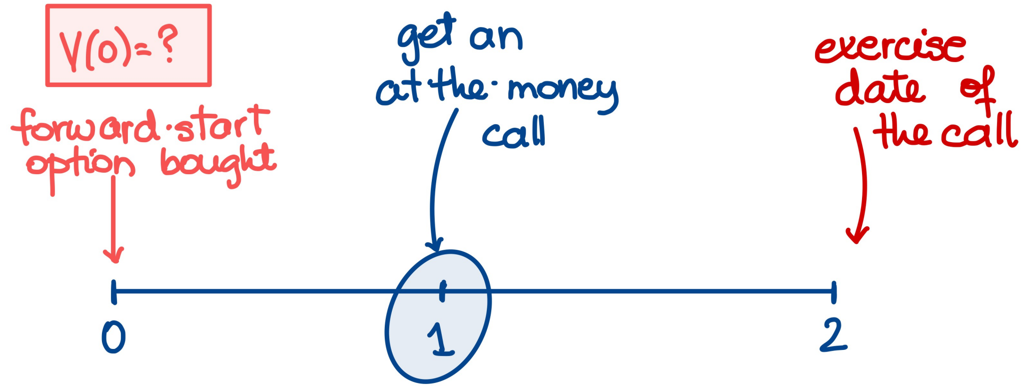
$$d_2 = d_1 - 0.5\sqrt{0.5} = \underline{-0.056}$$

$\text{pnorm}(0.2976) = \underline{0.383 = N(-d_1)}$

$\text{pnorm}(0.056) = \underline{0.5223 = N(-d_2)}$

$$V_p(0) = 98e^{-0.055(0.5)} \cdot (0.5223) - 100e^{-0.01(0.5)} \cdot (0.383)$$

$$= \underline{11.69}$$



(A) 586

(B) 594

(C) 684

(D) 692

(E) 797

19. Consider a *forward start option* which, 1 year from today, will give its owner a 1-year European call option with a strike price equal to the stock price at that time.

You are given:

(i) The European call option is on a stock that pays no dividends. $\boxed{\delta=0}$

(ii) The stock's volatility is 30%. $\boxed{\sigma=0.30}$

(iii) The forward price for delivery of 1 share of the stock 1 year from today is 100. $F_{0,T}=F_{0,1}(S)=100$

(iv) The continuously compounded risk-free interest rate is 8%. $r=0.08$

Under the Black-Scholes framework, determine the price today of the forward start option.

(A) 11.90

(B) 13.10

(C) 14.50

(D) 15.70

(E) 16.80

At $t<T$:

$$V_c(t)=S(t)e^{-\delta(T-t)}\cdot N(d_1) - Ke^{-r(T-t)}\cdot N(d_2)$$

w/ $d_1=\frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S(t)}{K}\right)+\left(r-\delta+\frac{\sigma^2}{2}\right)(T-t)\right]$

and $d_2=d_1-\sigma\sqrt{T-t}$

In this problem:

$$V_c(1)=S(1)\cdot N(d_1) - S(1)e^{-r(2-1)}\cdot N(d_2)$$

$$\boxed{V_c(1)=S(1)\left(N(d_1)-e^{-r}\cdot N(d_2)\right)}$$

$$d_1=\frac{1}{0.3\sqrt{2-1}}\left[\cancel{\ln\left(\frac{S(1)}{S(1)}\right)}+\left(0.08+\frac{0.09}{2}\right)(2-1)\right]=\frac{0.08+0.045}{0.3}$$

$$=0.4167$$

$$d_2 = 0.4167 - 0.3\sqrt{2-1} = 0.1167$$

$$N(d_1) = \text{pnorm}(0.4167) = 0.66155$$

$$N(d_2) = \text{pnorm}(0.1167) = 0.54645$$

$$V_c(1) = S(1)\cdot\left(\underline{0.66155 - e^{-0.08}\cdot 0.54645}\right) = S(1)\cdot 0.15711$$

At time 0, our forward start option is worth

$$0.15711\cdot F^P_{0,1}(S)$$

The answer is: $0.15711\cdot e^{-0.08}\cdot 100 = \underline{14.50}$ ■