M339W: February 18th, 2022.

Problem. Let the current stock price be \$100. Assume the Black-Scholes model. You're given:

(i) • $\mathbb{P}[S(1/4) < 95] = 0.2358$

(ii) • $\mathbb{P}[S(1/2) < 110] = 0.6026$

What's the expected time-1 stock price?

$\longrightarrow$: $\boxed{\mathbb{E}[S(T)] = S(0)e^{(\alpha-\delta)\cdot T}}$

In the B-S model:

$$S(T) = S(0)e^{(\alpha-\delta-\frac{\sigma^2}{2})\cdot T + \sigma\sqrt{T}Z}$$

$(\alpha-\delta-\frac{\sigma^2}{2}) =: \mu$

In particular:

$$\mathbb{E}[S(1)] = S(0)e^{(\alpha-\delta)} = \boxed{S(0)e^{\mu+\frac{\sigma^2}{2}}}$$

(i): 95 is the 23.58th quantile of $S(1/4)$

The 23.58th quantile of $N(0,1)$: qnorm(0.2358) = -0.72

$$95 = 100e^{\mu(1/4) + \sigma\sqrt{1/4}(-0.72)} \quad /:100$$

$$\ln \Big| \quad e^{\mu(1/4) + \sigma(1/2)(-0.72)} = 0.95$$

$$0.25\mu - 0.36\sigma = \ln(0.95) \quad \text{(i)} \quad /\cdot 2$$

(ii): 110 is the 60.26th quantile of $S(1/2)$

The 60.26th quantile of $N(0,1)$: qnorm(0.6026) = 0.26

$$110 = 100\,e^{\mu(1/2) + \sigma\sqrt{1/2}(0.26)} \quad /:100$$

$$\ln \Big| \quad e^{\mu(1/2) + \sigma\sqrt{1/2}(0.26)} = 1.1$$

$$0.5\mu + \sigma\sqrt{0.5}(0.26) = \ln(1.1) \quad \text{(ii)}$$

We solve for μ and σ in the system (i) & (ii):

$$\mu = \underline{\quad ? \quad} \quad \text{and} \quad \sigma = \underline{\quad ? \quad}$$

$$2\cdot(\text{i}): \cancel{0.5\,\mu} - 0.72\sigma = 2\ln(0.95)$$

$$(\text{ii}): \cancel{0.5\,\mu} + \sqrt{0.5}\,(0.26)\cdot\sigma = \ln(1.1)$$

$$\left(0.72 + \sqrt{0.5}\,(0.26)\right)\sigma = \ln(1.1) - 2\ln(0.95)$$

$$\underline{\sigma = R = 0.2189492} \quad \checkmark$$

$$4\cdot(\text{i}): \quad \mu - 1.44\cdot(0.2189) = 4\cdot\ln(0.95) \quad \checkmark$$

$$\underline{\mu = 0.110114}$$

Finally,
$$\mathbb{E}[S(1)] = 100\, e^{0.1101 + \frac{(0.2189)^2}{2}} = 114.35$$

Value@Risk [Review].

Start w/ a (small) probability p.

R... random variable corresponding to your return, or profit, or wealth

In this context, the $VaR_p(R)$ satisfies

$$\boxed{\mathbb{P}[R \leq VaR_p(R)] = p}$$

Problem. [Sample IFM : Part II : Problem #35]

Assume the Black-Scholes model.

You own one share of non-dividend-paying stock.
You intend to hold onto this stock for 4 years.

You want to set aside an amount of capital, in terms of a percentage of the initial asset price so that you reduce the risk of loss @ the end of the investment period.

- The mean rate of return on the stock is 0.15.
- The stock's volatility is 0.40.
- The VaR @ the 3rd quantile for the total portfolio equals the initial stock price.

Find the amount of capital to be put in reserve as a percentage of the initial stock price.

$\longrightarrow$:

$$\mathrm{VaR}_{0.03}(S(T) + \underbrace{C}_{=\varphi\cdot S(0)\ \ldots\text{ the amt of capital in reserve}}) = S(0)$$

By def'n, $\mathbb{P}[S(T) + \varphi\cdot S(0) < S(0)] = 0.03$

B-S Model: $S(T) = S(0)e^{(\alpha - \cancel{\delta} - \frac{\sigma^2}{2})\cdot T + \sigma\sqrt{T}\cdot Z}$

$$\mathbb{P}\left[\cancel{S(0)}e^{(\alpha-\frac{\sigma^2}{2})\cdot T + \sigma\sqrt{T}\cdot Z} + \varphi\cdot\cancel{S(0)} < \cancel{S(0)}\right] = 0.03$$

$$\mathbb{P}\left[\underbrace{e^{(\alpha-\frac{\sigma^2}{2})\cdot T + \sigma\sqrt{T}\cdot Z}} < 1-\varphi\right] = 0.03$$

The 3rd quantile of

qnorm(0.03) = −1.880794

$$e^{(0.15-\frac{0.16}{2})\cdot 4 + 0.4\sqrt{4}\cdot(-1.88)} = 1-\varphi$$

$$\boxed{\varphi = 0.7059}$$ ■