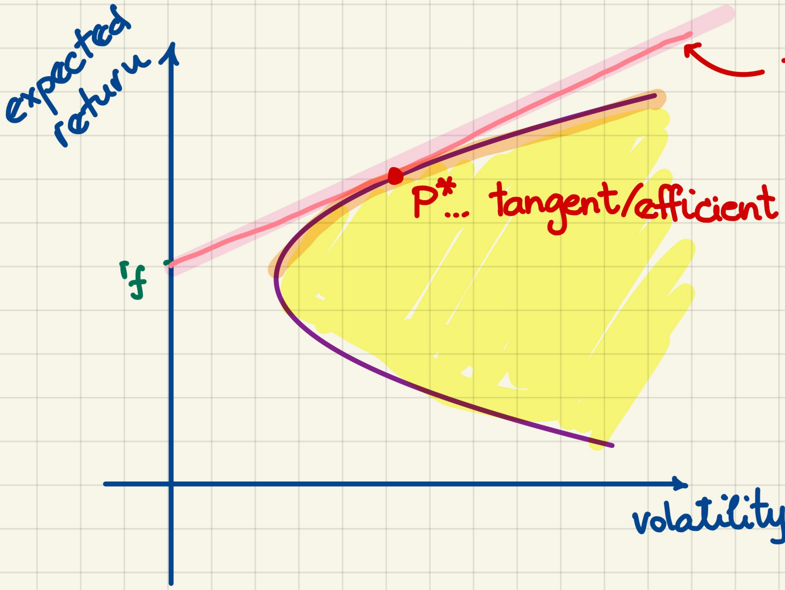
M339W: April 15th, 2022.

CAPM [cont'd].



The Capital Market Line (CML)

P^*... tangent/efficient portfolio = **MARKET PORTFOLIO**

All the assets i in the market:

$\times \begin{cases} \bullet \ \# \text{ of shares of } i \\ \bullet \ \text{market value of asset } i \text{ per share} \end{cases}$

MV_i = market value of asset i

In the market portfolio,

$$w_i := \frac{MV_i}{\sum_i MV_i}$$

In CAPM:

$$\boxed{\begin{aligned} &\mathbb{E}[R_I] = r_I = r_f + \underbrace{\beta_I}_{= \beta_I^{Mkt}} \left(\mathbb{E}[R_{Mkt}] - r_f\right) \\ &\text{w/} \quad \beta_I = \frac{\sigma_I}{\sigma_{Mkt}} \cdot \rho_{I,Mkt} = \frac{Cov[R_I, R_{Mkt}]}{Var[R_{Mkt}]} \end{aligned}}$$

15) You are given the following information about Stock X, Stock Y, and the market:

(i) The expected return and volatility for Stock X, Stock Y, and the market are shown in the table below:

	Required Return	Volatility
Stock X	3.0%	50%
Stock Y	?	35%
Market	6.0%	25%

(ii) The correlation between the returns of stock X and the market is –0.25.

(iii) The correlation between the returns of stock Y and the market is 0.30.

Assume the Capital Asset Pricing Model holds. Calculate the required return for Stock Y.

$$\beta_X = \frac{\sigma_X}{\sigma_{Mkt}} \cdot \rho_{X,Mkt} = \frac{0.5}{\cancel{0.25}}(-\cancel{0.25}) = -0.5 \checkmark$$

$$\beta_Y = \frac{\sigma_Y}{\sigma_{Mkt}} \cdot \rho_{Y,Mkt} = \frac{0.35}{0.25}(0.30) = \boxed{0.42} \checkmark$$

(A) 1.48%

(B) 2.52%

(C) 3.16%

(D) 4.84%

(E) 6.52%

$$r_X = r_f + \beta_X(\mathbb{E}[R_{Mkt}] - r_f)$$

$$0.03 = r_f + (-0.5)(0.06 - r_f) = r_f - 0.03 + 0.5 r_f$$

$$1.5 r_f = 0.06 \Rightarrow r_f = 0.04$$

$$r_Y = r_f + \beta_Y(\mathbb{E}[R_{Mkt}] - r_f) = 0.04 + 0.42(0.06 - 0.04) = 0.0484$$

$\square$

Beta of Portfolio.

Let P be a portfolio such that

$$R_P = w_1 \cdot R_1 + w_2 \cdot R_2 + \dots + w_n \cdot R_n$$

$$\beta_P = \frac{\sigma_P}{\sigma_{Mkt}} \cdot \rho_{P,Mkt} = \frac{Cov[R_P, R_{Mkt}]}{Var[R_{Mkt}]} =$$

$$= \frac{Cov[w_1 R_1 + \dots + w_n \cdot R_n, R_{Mkt}]}{Var[R_{Mkt}]} =$$

$$= \sum_{i=1}^{n} w_i \cdot \underbrace{\boxed{\frac{Cov[R_i, R_{Mkt}]}{Var[R_{Mkt}]}}}_{= \beta_i} = \sum_{i=1}^{n} w_i \cdot \beta_i$$

7) Consider a portfolio of four stocks as displayed in the following table:

Stock	Weight	Beta
1	0.1	1.3
2	0.2	-0.6
3	0.3	β_3
4	0.4	1.1

Assume the expected return of the portfolio is 0.12, the annual effective risk-free rate is 0.05, and the market risk premium is 0.08.

Assuming the Capital Asset Pricing Model holds, calculate β_3.

$$: \beta_P = w_1\beta_1 + w_2\beta_2 + w_3\beta_3 + w_4\beta_4$$

A) 0.80

B) 1.06

C) 1.42

D) 1.83

E) 2.17

$$\mathbb{E}[R_P] = r_P = r_f + \beta_P\left(\mathbb{E}[R_{Mkt}] - r_f\right)$$

$$0.12 = 0.05 + \beta_P(0.08)$$

$$\beta_P = \frac{0.12-0.05}{0.08} = 0.875$$

$$0.875 = 0.1(1.3) + 0.2(-0.6) + 0.3\beta_3 + 0.4(1.1)$$

$$\beta_3 = \frac{0.875 - 0.13 + 0.12 - 0.44}{0.3} = 1.4167 \quad \square$$

Task: Compare what we did to the "official" solution!

The Equity Cost of Capital.

In CAPM, for all i:

independent from i

$$\mathbb{E}[R_I] = r_I = \underbrace{r_f}_{\text{intercept}} + \beta_I \underbrace{\overbrace{(\mathbb{E}[R_{Mkt}] - r_f)}^{\text{independent from } i}}_{\text{slope}}$$

"independent argument"