M339W: April 13th, 2022.

Required Returns.

Objective: To figure out whether a portfolio P can be improved by "adding" (more of) a particular security I.

The Criterion:

$$\boxed{\mathbb{E}[R_I] > r_f + \underbrace{\boxed{\frac{\sigma_I}{\sigma_P} \cdot \rho_{P,I}}}_{=\ \beta_I^P} \left(\mathbb{E}[R_P] - r_f\right)}$$

$\boxed{\beta_I^P}$ ... the beta of the investment I w/ portfolio P

Def'n. The required return of investment I given portfolio P:

$$\boxed{r_I := r_f + \beta_I^P \left(\mathbb{E}[R_P] - r_f\right)}$$

Important Consequence:

Recall: A portfolio P^* is said to be efficient if no other portfolio outperforms it (in the sense of the Sharpe ratio).

Imagine an investment I such that

$$\mathbb{E}[R_I] \overset{X}{>} r_I = r_f + \beta_I^{P^*}\left(\mathbb{E}[R_{P^*}] - r_f\right)$$

$\Rightarrow$ Portfolio P^* can be improved by investing in I.

$\Rightarrow \Leftarrow$ Contradicts the fact that P^* is efficient.

$\Rightarrow$ For any security I:

$$\boxed{\mathbb{E}[R_I] = r_f + \beta_I^{*}\left(\mathbb{E}[R_P] - r_f\right)}$$

the beta of investment I w/ the efficient portfolio P^*

16) You are given the following information about Stock X and the ~~market~~: P

(i) The annual effective risk-free rate is 5%. $\boxed{r_f = 0.05}$

(ii) The expected return and volatility for Stock X and the market are shown in the table below:

	Expected Return	Volatility
Stock X	$\boxed{5\%}$	40%
P ~~Market~~	8%	25%

(iii) The correlation between the returns of stock X and the ~~market~~ P is $\boxed{-0.25.}$

~~Assume the Capital Asset Pricing Model holds~~. Calculate the required return for Stock X and determine if the investor should invest in Stock X.

X (A) The required return is 1.8%, and the investor should invest in Stock X.

(B) The required return is 3.8%, and the investor should NOT invest in stock X.

(C) The required return is 3.8%, and the investor should invest in stock X.

X (D) The required return is 6.2%, and the investor should NOT invest in Stock X.

X (E) The required return is 6.2%, and the investor should invest in stock X.

$$\beta_X^P = \frac{\sigma_X}{\sigma_P} \cdot \rho_{P,X} = \frac{0.40}{\cancel{0.25}}(-\cancel{0.25}) = -0.40$$

$$r_X = r_f + \beta_X^P\left(\mathbb{E}[R_P] - r_f\right) = 0.05 + (-0.40)(0.08-0.05) = 0.038$$

$$\mathbb{E}[R_X] = 0.05 > 0.038 = r_X$$

14) You are given the following information about Stock X, Stock Y, and the ~~market:~~ P

(i) The annual effective risk-free rate is 4%. $\boxed{r_f = 0.04}$

(ii) The expected return and volatility for Stock X, Stock Y, and the ~~market~~ P are shown in the table below:

	Expected Return	Volatility
Stock X	5.5%	40%
Stock Y	4.5%	35%
P ~~Market~~	6.0%	25%

(iii) The correlation between the returns of stock X and the ~~market~~ P is –0.25.

(iv) The correlation between the returns of stock Y and the ~~market~~ P is 0.30.

~~Assume the Capital Asset Pricing Model holds~~. Calculate the required returns for Stock X and Stock Y, and determine which of the two stocks an investor should choose.

(A) The required return for Stock X is 3.20%, the required return for Stock Y is 4.84%, and the investor should choose Stock X.

(B) The required return for Stock X is 3.20%, the required return for Stock Y is 4.84%, and the investor should choose Stock Y.

(C) The required return for Stock X is 4.80%, the required return for Stock Y is 4.84%, and the investor should choose Stock X.

(D) The required return for Stock X is 6.40%, the required return for Stock Y is 3.16%, and the investor should choose Stock Y.

(E) The required return for Stock X is 3.50%, the required return for Stock Y is 3.16%, and the investor should choose both Stock X and Stock Y.

$$\beta_x^P = \frac{\sigma_x}{\sigma_P} \cdot \rho_{P,x} = \frac{0.4}{\cancel{0.25}}(-\cancel{0.25}) = -0.40$$

$$r_x = 0.04 + (-0.40)(0.06 - 0.04) = 0.04 - 0.008 = \underline{0.032}$$

Since $E[R_x] = 0.05 > 0.032 = r_x$, we should invest in X.

Let's calculate what happens w/ Y.

$$\beta_Y^P = \frac{\sigma_Y}{\sigma_P} \rho_{P,Y} = \frac{0.35}{0.25}(0.3) = 0.42$$

$$r_Y = r_f + \beta_Y^P \left(\mathbb{E}[R_P] - r_f\right) = 0.04 + 0.42\,(0.06 - 0.04) = 0.0484$$

Comparison: $\mathbb{E}[R_Y] = 0.045 < 0.0484 = r_Y$

We should not invest in Y.

The Capital Asset Pricing Model (CAPM).

1. **No friction:** The investors buy/sell all the securities @ competitive market prices w/ no transaction costs. Both borrowing and lending are @ the same risk-free interest rate.

2. **Rationality:** Investors hold only efficient portfolios, i.e., portfolios which yield the highest possible expected return for a particular volatility.

3. **Homogeneous Expectations:**

All the investors have homogeneous beliefs about:

- expected returns
- volatilities
- correlation coefficients