M 339 W: April 11th, 2022.

Sharpe Ratio.

Start w/ a portfolio P consiting of risky investments.
Let $\boxed{R_P}$ denote its return.
Let $\boxed{r_f}$ denote the risk-free interest rate.

Now, we construct the portfolio xP so that:

- we give the weight x to the risky portfolio P

and

- we give the weight $(1-x)$ to the riskfree investment.

Let R_{xP} denote the return of this new portfolio.

We know: $\boxed{R_{xP} = x \cdot R_P + (1-x) \cdot r_f}$

- $\mathbb{E}[R_{xP}] = x \cdot \mathbb{E}[R_P] + (1-x) \cdot r_f$

$$= r_f + x\left(\mathbb{E}[R_P] - r_f\right)$$

$$\Rightarrow \boxed{\mathbb{E}[R_{xP}] - r_f = x\left(\mathbb{E}[R_P] - r_f\right)}$$

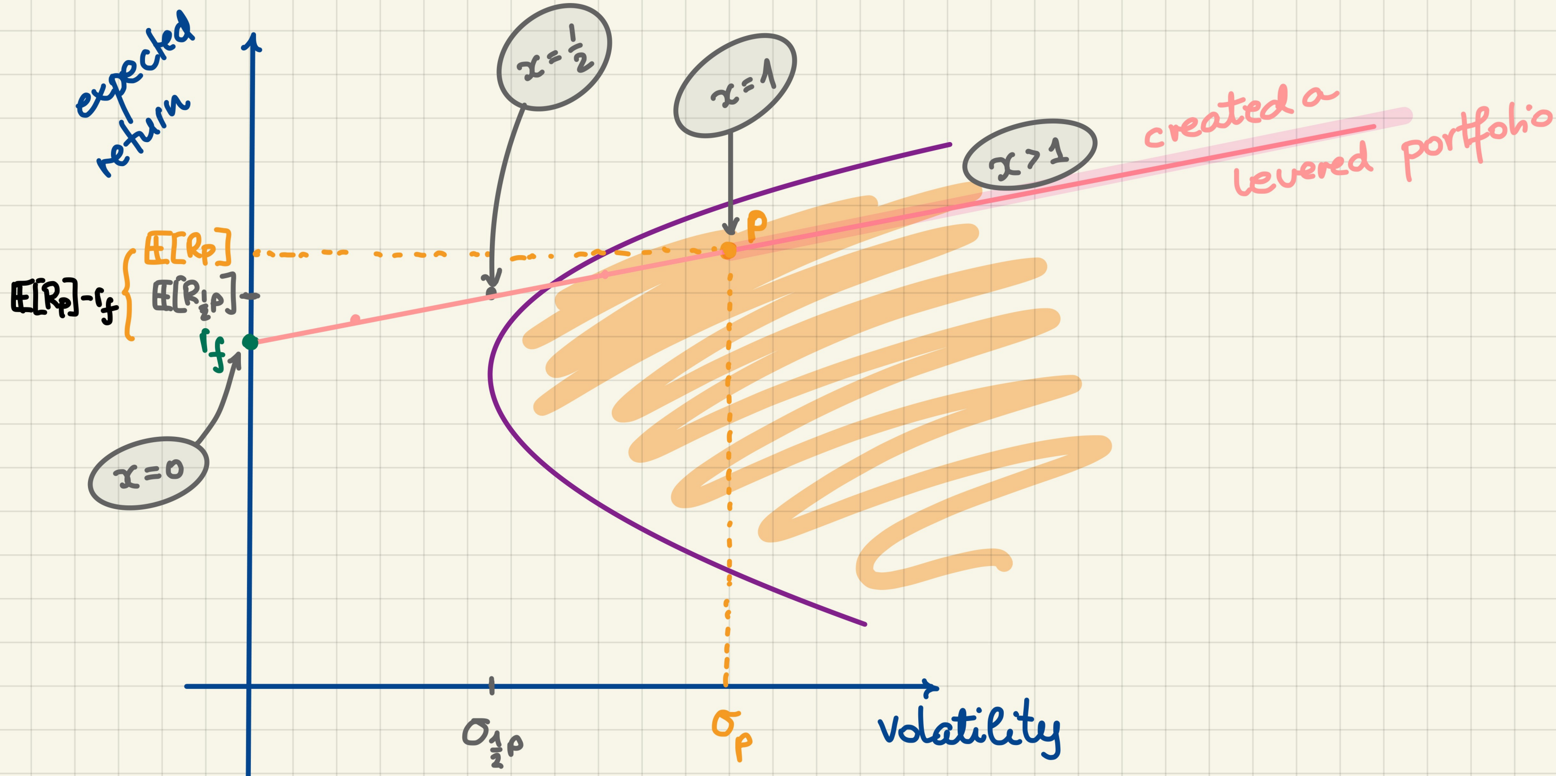
(expected) excess return
or risk premium

- $\mathrm{Var}[R_{xP}] = \mathrm{Var}[x \cdot R_P + \underbrace{(1-x) \cdot r_f}_{\text{deterministic}}]$

$$= \mathrm{Var}[x R_P]$$

$$= x^2 \cdot \mathrm{Var}[R_P]$$

$$\Rightarrow \boxed{SD[R_{xP}] = x \cdot SD[R_P]} \quad ✓$$

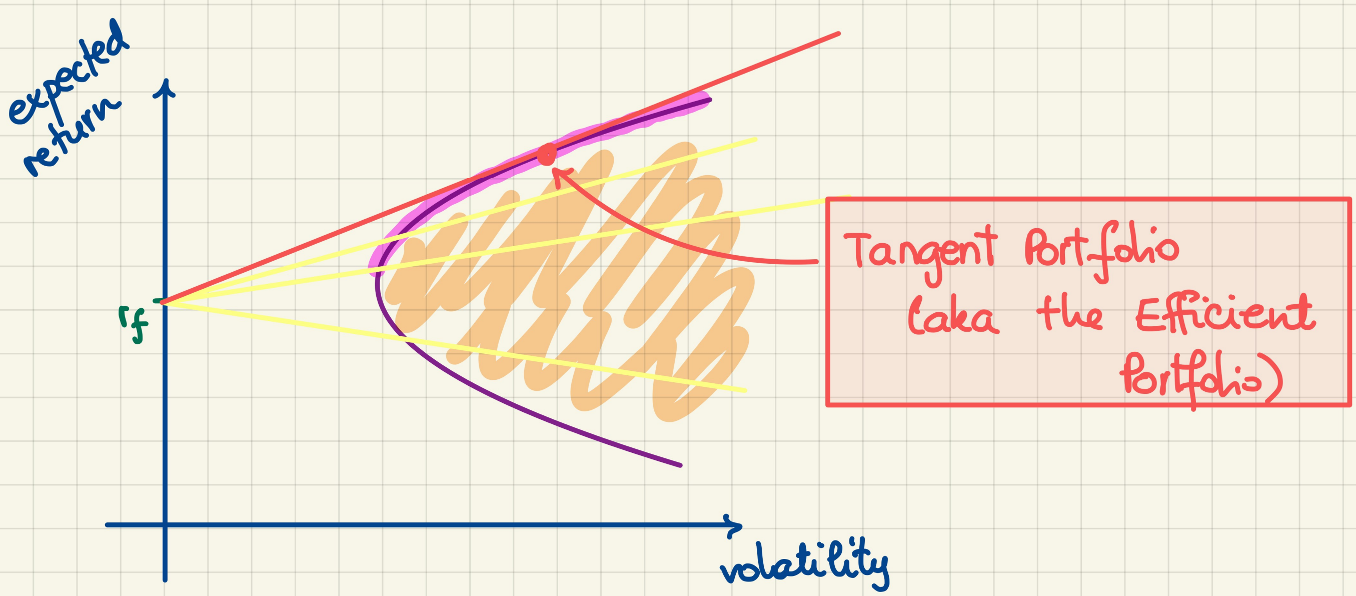


Q: What is the slope of the line through $(0, r_f)$ and $(\sigma_P, E[R_P])$?

$\longrightarrow$: $\text{slope} = \dfrac{\text{rise}}{\text{run}} = \boxed{\dfrac{E[R_P]-r_f}{\sigma_P}}$ Reward-to-Risk Ratio

THE SHARPE RATIO

Note:
- All the portfolios on the pink line above have the same Sharpe ratio.
- Higher Sharpe Ratios are preferable.



8) You are given the following information about a two-asset portfolio:

(i) $\boxed{\text{The Sharpe ratio of the portfolio is 0.3667.}}$

(ii) The annual effective risk-free rate is 4%. $r_f = 0.04$

$\boxed{\text{(iii)}}$ If the portfolio were 50% invested in a risk-free asset and 50% invested in a risky asset X, its expected return would be 9.50%. } P

Now, assume that the weights were revised so that the portfolio were 20% invested in a risk-free asset and 80% invested in risky asset X. } P'

Calculate the standard deviation of the portfolio return with the revised weights.

$\sigma_{P'} = ?$

(A) 6.0%

(B) 6.2%

(C) 12.8%

(D) 15.0%

(E) 24.0%

$\rightarrow$: $R_{P'} = 0.8 R_X + 0.2 r_f$

$\Rightarrow \boxed{\sigma_{P'} = 0.8 \cdot \sigma_X}$ ✓

(i) $\Rightarrow$ Sharpe ratio of X is 0.3667

$\Rightarrow$ By def'n.

$$\frac{\boxed{\mathbb{E}[R_X] - r_f}}{\sigma_X} = 0.3667$$

$$\frac{1}{2}\mathbb{E}[R_X] + \frac{1}{2} r_f = 0.095 \quad / \cdot 2$$

$$\mathbb{E}[R_X] + r_f = 0.19 \quad / (-2 r_f)$$

$$\mathbb{E}[R_X] - r_f = 0.19 - 2 r_f = 0.19 - 2(0.04)$$

$$= 0.19 - 0.08 = \boxed{0.11}$$

$$\sigma_X = \frac{0.11}{0.3667} = 0.3 \Rightarrow \boxed{\sigma_{P'} = 0.8 \cdot 0.3 = 0.24}$$