M339W: April 4th, 2022.

4) You are given the following information about a portfolio consisting of stocks X, Y, and Z:

Stock	Investment	Expected Return
X	10,000	$\boxed{8\%}$
Y	15,000	$\boxed{12\%}$
Z	25,000	$\boxed{16\%}$

$\Sigma = 50{,}000$

Calculate the expected return of the portfolio.

$\rightarrow$: R_P... return of the total portfolio

$\mathbb{E}[R_P] = ?$

(A) 10.8%

(B) 11.4%

(C) 12.0%

(D) 12.6%

(E) 13.2%

$$\mathbb{E}[R_P] = w_X \cdot \mathbb{E}[R_X] + w_Y \cdot \mathbb{E}[R_Y] + w_Z\, \mathbb{E}[R_Z]$$

$$w_X = \frac{10{,}000}{50{,}000} = 0.2$$

$$w_Y = \frac{15{,}000}{50{,}000} = 0.3$$

$$w_Z = 0.5$$

$$\mathbb{E}[R_P] = 0.2(0.08) + 0.3(0.12) + 0.5(0.16) = 0.132 \blacksquare$$

The Volatility of a Two-Stock Portfolio.

We index the two securities in the portfolio by $i = 1, 2$.

$$\boxed{R_P = w_1 \cdot R_1 + w_2 \cdot R_2} \quad \checkmark$$

$$\Rightarrow \boxed{\mathbb{E}[R_P] = w_1 \cdot \mathbb{E}[R_1] + w_2 \cdot \mathbb{E}[R_2]}$$

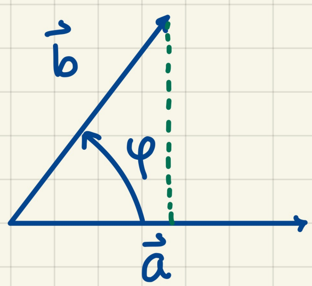
$$\boxed{Var[R_P]} = Var[w_1 \cdot R_1 + w_2 \cdot R_2]$$

$$= w_1^2 \, Var[R_1] + w_2^2 \cdot Var[R_2] + 2 w_1 w_2 \cdot \underline{Cov[R_1, R_2]}$$

By def'n:

$$Cov[R_1, R_2] = SD[R_1] \cdot SD[R_2] \cdot corr(R_1, R_2)$$

$$= \sigma_1 \cdot \sigma_2 \cdot \rho_{1,2}$$



scalar product:

$$\langle \vec{a}, \vec{b} \rangle = \vec{a} \cdot \vec{b}$$

$$= \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos(\varphi)$$

The volatility of the portfolio:

$$\boxed{\sigma_P = SD[R_P] = \sqrt{Var[R_P]}} \quad \checkmark$$

3) You are given the following information about the annual returns of two stocks, X and Y:

i) The expected returns of X and Y are $E[R_X] = \boxed{10\%}$ and $E[R_Y] = \boxed{15\%}$.

ii) The volatilities of the returns are $\cancel{V}_X = 18\%$ and $\cancel{V}_Y = 20\%$.

σ_X σ_Y

iii) The correlation coefficient of the returns for these two stocks is $\boxed{0.25}$.

iv) The expected return for a certain portfolio, consisting only of stocks X and Y, is 12%.

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Calculate the $\boxed{\text{volatility}}$ of the portfolio return.

$\longrightarrow$: $\sigma_p = \sqrt{Var[R_p]}$

(A) 10.88%

(B) 12.56%

(C) 13.55%

(D) 14.96%

(E) 16.91%

$$Var[R_p] = w_X^2 \cdot \sigma_X^2 + w_Y^2 \cdot \sigma_Y^2 + 2\, w_X \cdot w_Y \cdot \sigma_X \sigma_Y \cdot \boxed{\rho_{X,Y}}$$

$$0.12 = w_X \cdot \mathbb{E}[R_X] + w_Y \mathbb{E}[R_Y]$$

$$0.12 = w_X (0.10) + \underbrace{w_Y}_{= 1 - w_X} (0.15)$$

$$0.12 = w_X (0.10 - 0.15) + 0.15$$

$$\underbrace{w_X = \frac{0.15 - 0.12}{0.05} = \frac{3}{5} = 0.6 \quad \Rightarrow \quad w_Y = 0.4}$$

$$Var[R_p] = (0.6)^2 \cdot (0.18)^2 + (0.4)^2 \cdot (0.2)^2 + 2(0.6)(0.4)(0.18)(0.2)(0.25)$$

$$= 0.022384 \quad \Rightarrow \quad \boxed{\sigma_p = 0.1496}$$

$\square$

Volatility of an n-component portfolio.

$$R_p = \boxed{w_1 \cdot R_1 + w_2 R_2 + \cdots + w_n \cdot R_n}$$

$$\Rightarrow \quad Var[R_p] = Cov[\boxed{R_p}, R_p]$$

$$= Cov[w_1 R_1 + w_2 \cdot R_2 + \cdots + w_n \cdot R_n, R_p]$$

$$= w_1 \cdot Cov[R_1, R_p] + w_2 \cdot Cov[R_2, R_p] + \cdots + w_n \cdot Cov[R_n, R_p]$$

$$= \sum_{i=1}^{n} w_i \cdot Cov[R_i, R_p] \quad \checkmark$$

2) You are given the following information about a portfolio with four assets.

Asset	Market Value of Asset	Covariance of asset's return with the portfolio return
I	40,000	0.15
II	20,000	-0.10
III	10,000	0.20
IV	30,000	-0.05

$\Sigma = 100{,}000$

Calculate the standard deviation of the portfolio return.

$\longrightarrow$: $w_I = 0.4,\ w_{II} = 0.2,\ w_{III} = 0.1,\ w_{IV} = 0.3$

$$Var[R_p] = 0.4(0.15) + 0.2(-0.10) + 0.1(0.2) + 0.3(-0.05)$$

$$= 0.045$$

$$\sigma_p = \sqrt{0.045} = 0.212$$

$\square$

(A) 4.50%

(B) 13.2%

(C) 20.0%

(D) 21.2%

(E) 44.7%

Example.

- Portfolio P w/ components X and Y w/ weights w_X and w_Y.
- Economy can be "good" or "bad" w/ probability p_g and p_b, resp.

	G	B
X	x_g	x_b
Y	y_g	y_b

$$R_P = w_X \cdot R_X + w_Y \cdot R_Y$$

$$\boxed{R_P \sim \begin{cases} \underline{w_X \cdot x_g + w_Y \cdot y_g} & \text{"good" econ. w/ probab. } p_g \\ \underline{w_X \cdot x_b + w_Y \cdot y_b} & \text{"bad" econ. w/ probab. } p_b \end{cases}}$$