M339J: March 22nd, 2023.

280. A compound Poisson claim distribution has $\lambda = 5$ and individual claim amounts distributed as follows:

x	$f_X(x)$
$\boxed{5}$	0.6
k	0.4

Where $k > 5$

The expected cost of an aggregate stop-loss insurance subject to a deductible of 5 is 28.03.

Calculate k. →:

$$\mathbb{E}[(S-5)_+] = 28.03$$

$$= \boxed{\mathbb{E}[S]} - \boxed{\mathbb{E}[S \wedge 5]}$$

(A) 6

(B) 7

(C) 8

(D) 9

(E) 10

• $\mathbb{E}[S] = \lambda \cdot \mathbb{E}[X]$

$$= 5(5\cdot(0.6) + k(0.4))$$

$$= 2k + 15$$

281. DELETED

• $\mathbb{E}[S \wedge 5] = ?$

Support$(S \wedge 5) = \{0, 5\}$

$$S \wedge 5 \sim \begin{cases} 0 & \text{w/ probab. } p_N(0) = e^{-5} \\ 5 & \text{w/ probab. } 1 - e^{-5} \end{cases}$$

$$\mathbb{E}[S \wedge 5] = 0 \cdot e^{-5} + 5 \cdot (1 - e^{-5}) = 5(1 - e^{-5})$$

$$28.03 = 2k + 15 - 5(1 - e^{-5})$$

$$k = \frac{18.03 - 5e^{-5}}{2} = 8.99$$

□

289. A compound Poisson distribution has $\boxed{\lambda = 5}$ and claim amount distribution as follows:

x	$p(x)$
100	0.80
500	0.16
1000	0.04

Calculate the probability that aggregate claims will be exactly 600.

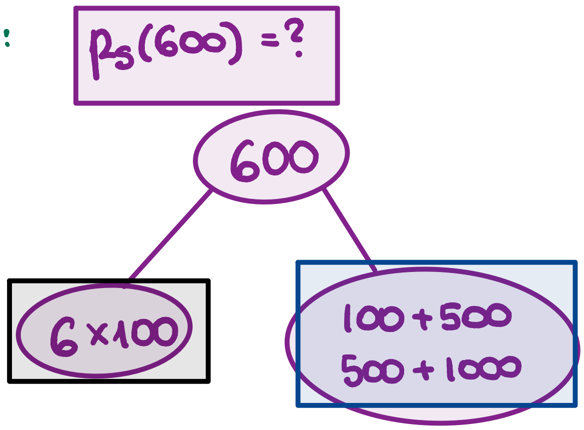
(A) 0.022

(B) 0.038

(C) 0.049

(D) 0.060

(E) 0.070



$\checkmark\ \mathbb{P}[6 \text{ claims of } 100] = \mathbb{P}[N=6, X_1 = X_2 = \dots = X_6 = 100]$

$= p_N(6) \cdot (p_X(100))^6 = e^{-5} \cdot \dfrac{5^6}{6!} (0.8)^6$

independence

$= \underline{0.03833}$

$\checkmark\ \mathbb{P}[2 \text{ claims, one is } 100, \text{ other is } 500] =$

$= 2 \cdot p_N(2) \cdot p_X(100) \cdot p_X(500) = \underline{0.02156}$

answer: $0.03833 + 0.02156 = 0.05989$

290. DELETED

291. DELETED

292 DELETED

293 DELETED

294 DELETED

295 DELETED

296. DELETED

297. DELETED

298. DELETED

299. DELETED

A Few Compound Poissons.

We start w/ n different streams of losses, each of them a compound Poissons and all independent.

$\{S_j, j=1..n\}$ are independent compound Poisson, i.e.,

for every j: $N_j \sim Poisson(\lambda_j)$

and $X^j \sim cdf\ F_j$

More precisely, for stream j, the severity r.v.s are
$\{X_1^j, X_2^j, \ldots, X_k^j, \ldots\}$

$$\boxed{S_j = X_1^j + X_2^j + \cdots + X_{N_j}^j}$$

Set: $\boxed{S = S_1 + S_2 + \cdots + S_n}$

Thm. S is itself compound Poisson, w/

$N = N_1 + N_2 + \cdots + N_n \sim Poisson(\lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_n)$

and w/ the severity r.v. w/ the cdf

$$\boxed{F_X(x) = \sum_{j=1}^{n} \frac{\lambda_j}{\lambda} \cdot F_j(x)}$$

n-point mixture of X^j's (M349P ☺)

125. Two types of insurance claims are made to an insurance company. For each type, the number of claims follows a Poisson distribution and the amount of each claim is uniformly distributed as follows:

Type of Claim	Poisson Parameter λ for Number of Claims in one year	Range of Each Claim Amount
I	12 $=\lambda_I$	(0, 1) $X^I \sim U(0,1)$
II	4 $=\lambda_{II}$	(0, 5) $X^{II} \sim U(0,5)$

The numbers of claims of the two types are independent and the claim amounts and claim numbers are independent.

Calculate the normal approximation to the probability that the total of claim amounts (S) in one year exceeds 18.

(A) 0.37

(B) 0.39

(C) 0.41

(D) 0.43

(E) 0.45

$\longrightarrow$:

$$S_I = X_1^I + X_2^I + \cdots + X_{N_I}^I$$

$$S_{II} = X_1^{II} + X_2^{II} + \cdots + X_{N_{II}}^{II}$$

$$\boxed{S = S_I + S_{II}}$$

$$\mathbb{P}[S>18] = ?$$

The normal approximation requires that we express S in standard units. So, we need $\boxed{\mathbb{E}[S]}$ and $\boxed{Var[S]}$.

$$\mathbb{E}[S] = \mathbb{E}[S_I] + \mathbb{E}[S_{II}] = 12\cdot\left(\tfrac{1}{2}\right) + 4\left(\tfrac{5}{2}\right) = 16$$

$$Var[S] = Var[S_I + S_{II}]$$ Independence!

$$= Var[S_I] + Var[S_{II}]$$

$$Var[S_I] = \mathbb{E}[N_I]\,Var[X^I] + Var[N_I]\left(\mathbb{E}[X^I]\right)^2$$

$$= 12 \cdot \frac{1}{12} + 12 \cdot \left(\tfrac{1}{2}\right)^2$$

$$= 1 + 3 = 4$$

$$\checkmark\ Var[S_{II}] = \mathbb{E}[N_{II}] \cdot Var[X^{II}] + Var[N_{II}] \cdot (\mathbb{E}[X^{II}])^2$$

$$= 4 \cdot \underline{\frac{25}{12}} + 4 \cdot \left(\frac{5}{2}\right)^2$$

$$= \frac{25}{3} + 25 = \frac{100}{3}$$

$$Var[S] = 4 + \frac{100}{3} = 37.333\dot{3} \implies SD[S] = \sigma_S = \underline{6.1101}$$

Method #1.

$$S \approx Normal(mean = 16,\ sd = 6.1101)$$

$$\mathbb{P}[S > 18] = 1 - pnorm(18, mean = 16,\ sd = sqrt(100/3)) =$$

$$= 0.3645172$$

Method #2.

$$\mathbb{P}[S > 18] = \mathbb{P}\left[\frac{S-16}{6.1101} > \frac{18-16}{6.1101}\right]$$

$\sim N(0,1) \sim Z$

$$= \mathbb{P}[Z > 0.33] = 1 - \Phi(0.33)$$

$$= 1 - 0.6293 = 0.3707 \quad \square$$

Problem. Medical and dental claims are assumed to be independent w/ compound Poisson dist'ns.

Claim Type	Poisson rate	Claim Amt Dist.
Medical	2	$U(0,1000) \sim X_M$
Dental	3	$U(0,200) \sim X_D$

Let X be a random variable which denotes a randomly chosen claim under a policy which covers both medical and dental claims.

Find $\boxed{\mathbb{E}[(X-100)_+]}$.

$\longrightarrow$: The combined claim count is $N \sim$ Poisson$(\lambda = 2+3 = 5)$.

For an individual claim amount X, its cdf is:

$$F_X(x) = \frac{2}{5} \cdot F_{X_M}(x) + \frac{3}{5} \cdot F_{X_D}(x)$$

$\Rightarrow$ its pdf:

$$f_X(x) = \frac{2}{5} \cdot f_{X_M}(x) + \frac{3}{5} f_{X_D}(x)$$

$$= \begin{cases} \frac{2}{5} \cdot \frac{1}{1000} + \frac{3}{5} \cdot \frac{1}{200} & x \in (0,200) \\ \frac{2}{5} \cdot \frac{1}{1000} & x \in (200,1000) \end{cases}$$

$$= \begin{cases} \frac{17}{5000} & x \in (0,200) \\ \frac{1}{2500} & x \in (200,1000) \end{cases}$$

$$\mathbb{E}[(X-100)_+] = \int_{100}^{1000} (x-100) f_X(x)\, dx$$

$$= \int_{100}^{1000} x f_X(x)\, dx - 100 \int_{100}^{1000} f_X(x)\, dx$$

$$\int_{100}^{1000} f_X(x)\, dx = 1 - \int_0^{100} f_X(x)\, dx = 1 - \frac{17}{5000} \cdot 100$$

$$= 1 - \frac{17}{50}$$

$$= \frac{33}{50}$$

$$= \int_{100}^{200} x\left(\frac{17}{5000}\right)dx + \int_{200}^{1000} x\left(\frac{1}{2500}\right)dx - 100 \cdot \frac{33}{50}$$

$$= \frac{17}{5000} \cdot \frac{x^2}{2} \Big|_{x=100}^{200} + \frac{1}{2500} \cdot \frac{x^2}{2} \Big|_{x=200}^{1000} - 66$$

$$= \underline{177} \quad \square$$