M339J : March 20th, 2023.

16. You are given:

		Mean	Standard Deviation
N	Number of Claims	8	3
X	Individual Losses	10,000	3,937

Using the normal approximation, determine the probability that the aggregate loss S will exceed 150% of the expected loss μ_S.

(A) $\Phi(1.25)$

(B) $\Phi(1.5)$

(C) $1-\Phi(1.25)$

(D) $1-\Phi(1.5)$

(E) $1.5\,\Phi(1)$

$\mathbb{P}[S > 1.5\mu_S] = ?$

- $\mu_S = \mathbb{E}[S] = \mathbb{E}[N]\cdot\mathbb{E}[X] = 80{,}000$
- $\sigma_S^2 = Var[S] = Var[X]\cdot\mathbb{E}[N] + Var[N]\cdot(\mathbb{E}[X])^2$

$= (3937)^2\cdot 8 + 3^2 \cdot (10000)^2$

$= 1023999752$

$\sigma_S = 32000$

$$\mathbb{P}[S > 1.5\mu_S] = \mathbb{P}\Big[\underbrace{\frac{S-\mu_S}{\sigma_S}}_{\approx N(0,1)\sim Z} > \frac{1.5\mu_S-\mu_S}{\sigma_S}\Big]$$

$$= \mathbb{P}\Big[Z > \frac{0.5\mu_S}{\sigma_S}\Big] = \mathbb{P}\Big[Z > \frac{40000}{32000}\Big]$$

$$= \mathbb{P}[Z > 1.25] = 1 - \mathbb{P}[Z \le 1.25]$$

$$= 1 - \Phi(1.25) \qquad \square$$

32. For an individual over 65:

$N \sim \mathbb{P}oisson(\lambda = 25)$

(i) The number of pharmacy claims is a Poisson random variable with mean 25.

$X \sim U(5,95)$

(ii) The amount of each pharmacy claim is uniformly distributed between 5 and 95.

(iii) The amounts of the claims and the number of claims are mutually independent.

Determine the probability that aggregate claims (S) for this individual will exceed 2000 using the
normal approximation.

$\mathbb{P}[S > 2000] = ?$

(A) $1 - \Phi(1.33)$

(B) $1 - \Phi(1.66)$

(C) $1 - \Phi(2.33)$

(D) $1 - \Phi(2.66)$

(E) $1 - \Phi(3.33)$

- $\mu_S = \mathbb{E}[S] = \mathbb{E}[N] \cdot \mathbb{E}[X] = 25 \cdot \frac{5+95}{2} = 1250$
- $\sigma_S^2 = Var[S] = \mathbb{E}[N] \cdot Var[X] + Var[N] \cdot (\mathbb{E}[X])^2$

$= 25\left(\frac{(95-5)^2}{12} + 50^2\right) = 79,375$

$\sigma_S = 281.736$

$\mathbb{P}[S > 2000] = \mathbb{P}\left[\underbrace{\frac{S-\mu_S}{\sigma_S}}_{Z \sim N(0,1) \sim Z} > \frac{2000-1250}{281.736}\right]$

$= \mathbb{P}[Z > 2.66] = 1 - \Phi(2.66)$ $\square$

"Def'n." Insurance on the aggregate losses subject to an ordinary deductible d is called **stop-loss insurance**. The expected cost of this type of insurance is called **net stop-loss premium**.

$$\boxed{\mathbb{E}\left[(S-d)_+\right]}$$

Note: The following are useful:

- the tail formula: $\mathbb{E}[(S-d)_+] = \int_d^{+\infty} (1-F_S(x))\,dx$
- $\mathbb{E}[(S-d)_+] = \mathbb{E}[S] - \boxed{\mathbb{E}[S\wedge d]}$
- combinatorics.