14. In modeling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that for all integers $n \geq 0$, $\boxed{p(n+1) = 0.2\,p(n)}$ where $p(n)$ represents the probability that the policyholder files n claims during the period.

Support
↳ $\mathbb{N}_0$

Under this assumption, calculate the probability that a policyholder files more than one claim during the period.

(A) 0.04
(B) 0.16
(C) 0.20
(D) 0.80
(E) 0.96

→ : N… total # of claims

$\mathbb{P}[N > 1] = x = 1 - \mathbb{P}[N=0] - \mathbb{P}[N=1]$

$= 1 - p_0 - p_1$

15. An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose coverages A, B, and C are 1/4, 1/3, and 5/12 respectively.

Calculate the probability that a randomly chosen employee will choose no supplementary coverage.

(A) 0
(B) 47/144
(C) 1/2
(D) 97/144
(E) 7/9

16. An insurance company determines that N, the number of claims received in a week, is a random variable with $P[N = n] = \dfrac{1}{2^{n+1}}$ where $n \geq 0$. The company also determines that the number of claims received in a given week is independent of the number of claims received in any other week.

Calculate the probability that exactly seven claims will be received during a given two-week period.

(A) 1/256
(B) 1/128
(C) 7/512
(D) 1/64
(E) 1/32

$\longrightarrow$:

p_0 ... prob. of zero claims

$\boxed{p_1 = 0.2 p_0}$

$p_2 = (0.2) p_1 = (0.2)^2 p_0$

$\vdots$

$p_k = (0.2)^k p_0$

$\vdots$

The pmf must sum to 1 :

$$1 = \sum_{k=0}^{+\infty} p_k = \sum_{k=0}^{+\infty} \left((0.2)^k p_0 \right) = p_0 \boxed{\sum_{k=0}^{+\infty} (0.2)^k}$$

$$1 = p_0 \cdot \frac{1}{1-0.2} = p_0 \cdot \frac{5}{4} \Rightarrow \boxed{p_0 = \frac{4}{5}}$$

$$\mathbb{P}[N>1] = 1 - p_0 - p_1 = 1 - \frac{4}{5} - \frac{1}{5} \cdot \frac{4}{5} = \frac{1}{25} = 0.04 \quad \square$$

Alternatively, look @ the tables w/ $a=0.2$ and $b=0$

Tables $\Rightarrow$ $a = \frac{\beta}{1+\beta} \Rightarrow \frac{1}{5} = \frac{\beta}{1+\beta}$

$$1+\beta = 5\beta$$

$$1 = 4\beta$$

$$\boxed{\beta = \frac{1}{4}}$$

$\Rightarrow N \sim \text{geometric}\left(\beta = \frac{1}{4}\right)$

$\Rightarrow p_0 = \frac{1}{1+\beta} = \frac{1}{1+\frac{1}{4}} = \frac{4}{5}$, i.e.,

$N \sim \text{geometric}(\text{prob. of success} = \frac{4}{5})$

$\Rightarrow p_1 = \frac{\beta^1}{(1+\beta)^2} = \frac{\frac{1}{4}}{\left(\frac{5}{4}\right)^2} = \frac{4}{25}$

$\Rightarrow$ answer :

$1 - \frac{4}{5} - \frac{4}{25} = \frac{1}{25}$

93. At the beginning of each round of a game of chance the player pays 12.5. The player then rolls one die with outcome N. The player then rolls N dice and wins an amount equal to the total of the numbers showing on the N dice. All dice have 6 sides and are fair.

Using the normal approximation, calculate the probability that a player starting with 15,000 will have at least 15,000 after 1000 rounds.

(A) 0.01

(B) 0.04

(C) 0.06

(D) 0.09

(E) 0.12

94. X is a discrete random variable with a probability function that is a member of the $(a,b,0)$ class of distributions.

You are given:

(i) $\Pr(X=0)=\Pr(X=1)=0.25$ $\boxed{p_0 = p_1 = 0.25}$

(ii) $\Pr(X=2)=0.1875$ $p_2 = 0.1875$

Calculate $\Pr(X=3)$.

(A) 0.120

(B) 0.125

(C) 0.130

(D) 0.135

(E) 0.140

(a,b,0) class: $p_k = p_{k-1}\left(a+\frac{b}{k}\right)$ $k=1,2,\ldots$

for some constants a and b

(i) $p_1 = p_0\left(a+\frac{b}{1}\right) \Rightarrow 0.25 = 0.25(a+b)$

$\Rightarrow \boxed{a+b=1}$ ✓

(ii) $p_2 = p_1\left(a+\frac{b}{2}\right) \Rightarrow 0.1875 = 0.25\left(a+\frac{b}{2}\right)$

$\Rightarrow \boxed{a+\frac{b}{2}=0.75}$ ✓

$\Rightarrow \frac{b}{2} = 0.25 \Rightarrow \boxed{b=0.5=a}$

STAM Tables:

$X \sim NegBinomial(r=2, \beta=1)$

Even w/out the tables:

$$p_3 = p_2\left(a+\frac{b}{3}\right) = 0.1875(0.5)\left(1+\frac{1}{3}\right) = 0.125$$

The Impact of Deductibles on Claim Frequency.

On Compounding.

INDEPENDENT

In general, for an $\mathbb{N}_0$-valued random variable N w/ the pgf P_N

and

a sequence of independent, identically dist'd random variables $\boxed{\{M_1, M_2, \dots\}}$ w/ a common pgf P_M,

we set

$$S = M_1 + M_2 + \cdots + M_N = \sum_{i=1}^{N} M_i$$

(If $N=0$, then $S=0$).

Q: What is the dist'n of S?

$\longrightarrow$: If N is independent from $\{M_1, M_2, \dots\}$,

then,
$$\boxed{P_S(z) = P_N(P_M(z))}$$

Example.
- N... # of accidents per year
- $\{M_j : j=1,2,\dots\}$ ... the # claims per accident (e.g., # of people in all cars)

- N... # of losses
- $\{M_j : j=1,2,\dots\}$ ... indicate whether the loss met the deductible or not
- S.... the number of claims