M339J: March 3rd, 2023.

Binomial Distribution.

Consider m independent, identically distributed risks w/ probability of making a claim is denoted by q.

Formally, for $j = 1, \dots, m$, we set

$$I_j = \begin{cases} 1 & \text{if risk } j \text{ makes a claim} \\ 0 & \text{if risk } j \text{ does not make a claim} \end{cases}$$

Then, for all j,

$$I_j \sim \text{Bernoulli}(q)$$

and $\{I_j, j = 1..m\}$ are independent.

N... the total number of claims made

$$\boxed{N = I_1 + I_2 + \dots + I_m}$$

Note: There are many approaches to this. I am choosing this one to showcase a specific technique.

First, figure out the pgf of N.

$$P_N(z) \underset{\text{Thm.}}{=} P_{I_1}(z) \cdot P_{I_2}(z) \cdot \dots \cdot P_{I_m}(z)$$

$$\underset{\substack{\text{identically}\\ \text{dist'd}}}{=} (P_{I_1}(z))^m$$

For a single Bernoulli trial w/ probab. of success q:

$$P_{I_1}(z) \underset{\text{by def'n.}}{=} \mathbb{E}[z^{I_1}] = \underbrace{p_{I_1}(0)}_{=1-q} z^0 + \underbrace{p_{I_1}(1)}_{=q} \cdot z^1 = (1-q) + q \cdot z$$

$$I_1 \sim \begin{cases} 0 \\ 1 \end{cases}$$

$$\boxed{P_{I_1}(z) = 1 + q(z-1)}$$

$$\boxed{P_N(z) = (1 + q(z-1))^m}$$

Using the binomial formula for $P_N(z) = ((1-q) + qz)^m$ we get the pmf of N.

287. For an aggregate loss distribution S:

(i) The number of claims has a negative binomial distribution with $r = 16$ and $\beta = 6$.

(ii) The claim amounts are uniformly distributed on the interval (0, 8).

(iii) The number of claims and claim amounts are mutually independent.

Using the normal approximation for aggregate losses, calculate the premium such that the probability that aggregate losses will exceed the premium is 5%.

(A) 500

(B) 520

(C) 540

(D) 560

Enrichment !

(E) 580

288. The random variable N has a mixed distribution:

(i) With probability p, N has a binomial distribution with $q = 0.5$ and $m = 2$.

(ii) With probability $1 - p$, N has a binomial distribution with $q = 0.5$ and $m = 4$.

Which of the following is a correct expression for $\Pr(N = 2)$?

(A) $0.125p^2$

(B) $0.375 + 0.125p$

(C) $0.375 + 0.125p^2$

(D) $0.375 - 0.125p^2$

(E) $0.375 - 0.125p$

Note on the "counting" dist'ns:

	mean		variance
Poisson(λ)	λ	$=$	λ
NegBinomial(r, β)	$r\beta$	$<$	$r\beta\underbrace{(1+\beta)}_{>1}$
Binomial(m, q)	$m \cdot q$	$>$	$mq\underbrace{(1-q)}_{<1}$

Poisson "thinning" & the binomial.

Let $N \sim$ Poisson(λ) be our frequency random variable.

Every loss is from:

- Category 1 w/ probab. p_1
- Category 2 w/ probab. p_2 w/ $p_1 + p_2 = 1$

N_i ... # of events from Category i, $i = 1, 2$.

From the "thinning thm", we know that:

- N_1 and N_2 are independent

and

- $N_i \sim$ Poisson($\lambda_i = p_i \cdot \lambda$), $i = 1, 2$

Q: Given that $N = m$, what is the probability that

$N_1 = k$ for $k = 0, 1, \ldots, m$?

$\rightarrow$:

$$\mathbb{P}[N_1 = k \mid N = m] \underset{\substack{\text{by the def'n} \\ \text{of conditional} \\ \text{probability}}}{=} \frac{\mathbb{P}[N_1 = k, N = m]}{\mathbb{P}[N = m]} = \frac{\mathbb{P}[N_1 = k, N_2 = m-k]}{\mathbb{P}[N = m]}$$

$$\underset{\substack{N_1 \text{ and } N_2 \\ \text{are independent}}}{=} \frac{\boxed{\mathbb{P}[N_1 = k]} \cdot \boxed{\mathbb{P}[N_2 = m-k]}}{\boxed{\mathbb{P}[N = m]}} \underset{\text{Poisson}}{=} \frac{\cancel{e^{-\lambda_1}} \cdot \frac{\lambda_1^k}{k!} \cdot \cancel{e^{-\lambda_2}} \cdot \frac{\lambda_2^{m-k}}{(m-k)!}}{\cancel{e^{-\lambda}} \cdot \frac{\lambda^m}{m!}} =$$

$$= \boxed{\frac{m!}{k!\,(m-k)!}} \cdot \frac{p_1^k \cdot \cancel{\lambda^k} \cdot p_2^{m-k} \cdot \cancel{\lambda^{m-k}}}{\cancel{\lambda^m}}$$

$$= \boxed{\binom{m}{k}} p_1^k \cdot p_2^{m-k}$$

$$\boxed{N_1 \mid N=m \sim \text{Binomial}(m, q=p_1)}$$

The (a,b,0) class.

If an $\mathbb{N}_0$-valued random variable has a pmf which satisfies the following recursion:

$$p_k = p_{k-1}\left(a + \frac{b}{k}\right) \quad \text{for all } k=1,2,\dots,$$

then, we say that is has an (a,b,0) class distribution.

The Poisson, NegBinomial, and Binomial are the only representatives.

14. In modeling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that for all integers $n \geq 0$, $p(n+1) = 0.2\,p(n)$ where $p(n)$ represents the probability that the policyholder files n claims during the period.

Under this assumption, calculate the probability that a policyholder files more than one claim during the period.

(A) 0.04
(B) 0.16
(C) 0.20
(D) 0.80
(E) 0.96

$\boxed{p_{n+1} = p_n(0.2)}$

N... total # of claims

$P[N>1] = \cancel{X} = 1 - P[N=0] - P[N=1] = \underline{1 - p_0 - p_1}$

15. An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose coverages A, B, and C are 1/4, 1/3, and 5/12 respectively.

Calculate the probability that a randomly chosen employee will choose no supplementary coverage.

(A) 0
(B) 47/144
(C) 1/2
(D) 97/144
(E) 7/9

16. An insurance company determines that N, the number of claims received in a week, is a random variable with $P[N = n] = \dfrac{1}{2^{n+1}}$ where $n \geq 0$. The company also determines that the number of claims received in a given week is independent of the number of claims received in any other week.

Calculate the probability that exactly seven claims will be received during a given two-week period.

(A) 1/256
(B) 1/128
(C) 7/512
(D) 1/64
(E) 1/32