M339J: February 8^{th}, 2023.

168. For an insurance:

(i) Losses can be 100, 200 or 300 with respective probabilities 0.2, 0.2, and 0.6.

(ii) The insurance has an ordinary deductible of 150 per loss.

(iii) Y^P is the claim payment per payment random variable.

Calculate $\mathrm{Var}(Y^P)$.

→: By def'n:

$Y^P = X - d \mid X > d$ w/ $d = 150$

losses X	100	200	300
Y^P	undef.	50	150

Support of Y^P

(A) 1500

(B) 1875

(C) 2250

(D) 2625

(E) 3000

169. The distribution of a loss, X, is a two-point mixture:

(i) With probability 0.8, X has a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 100$.

(ii) With probability 0.2, X has a two-parameter Pareto distribution with with $\alpha = 4$ and $\theta = 3000$.

Calculate $\Pr(X \le 200)$.

(A) 0.76

(B) 0.79

(C) 0.82

(D) 0.85

(E) 0.88

Method I:

$$Var[Y^P] = \mathbb{E}[(Y^P)^2] - (\mathbb{E}[Y^P])^2$$

$\boxed{p_{Y^P}}$ ... the probability mass f'tion of Y^P

$$p_{Y^P}(50) = \frac{p_X(200)}{S_X(150)} = \frac{0.2}{0.2+0.6} = 0.25$$

$$p_{Y^P}(150) = \frac{p_X(300)}{S_X(150)} = \frac{0.6}{0.2+0.6} = 0.75$$

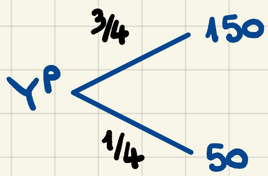
$$\mathbb{E}[Y^P] = 0.25(50) + 0.75(150) = \underline{125}$$

$$\mathbb{E}[(Y^P)^2] = 0.25(50)^2 + 0.75(150)^2 = \underline{17{,}500}$$

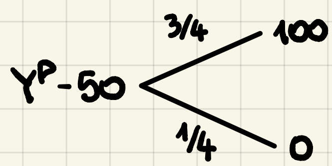
$$Var[Y^P] = 17500 - (125)^2 = \underline{1875}$$

□

Method II:



$$Var[Y^P] = Var[Y^P - 50]$$



$Y^P - 50 \sim 100 \cdot \text{Bernoulli}(\text{prob. of success} = 3/4)$

$$\Rightarrow Var[Y^P - 50] = 100^2 \cdot \left(\tfrac{3}{4}\right)\left(\tfrac{1}{4}\right) = 1875$$

□

13. The loss severity random variable X follows the exponential distribution with mean 10,000.

Determine the coefficient of variation of the ~~excess loss variable~~ $Y = \max(X - 30000, 0)$.

(A) 1.0

(B) 3.0

(C) 6.3

(D) 9.0

(E) 39.2

$$\boxed{\frac{\sigma_Y}{\mu_Y}}$$

Steps: 1. μ_Y 2. $\mathbb{E}[Y^2] = ?$ $\Big\}$ 3. $\longrightarrow \sigma_Y = \sqrt{Var[Y]}$

$$Y^L = \begin{cases} Y^P \ (x > d) & \text{w/ probab. } S_X(d) \\ 0 \ (x \le d) & \text{w/ probab. } F_X(d) \end{cases}$$

In this problem:

By the memoryless property: $Y^P \sim \text{Exponential}(\theta)$

1. $\mu_Y = \cancel{X}\ \mathbb{E}[Y^L] = \underbrace{\mathbb{E}[Y^P]}\cdot S_X(d) + 0 \cdot \cancel{F_X(d)}$
$= \theta \cdot e^{-\frac{d}{\theta}}$

2. $\mathbb{E}[Y^2] = \mathbb{E}[(Y^P)^2] \cdot S_X(d) + 0^2 \cdot \cancel{F_X(d)}$
$= 2\theta^2 \cdot e^{-\frac{d}{\theta}}$

3. $Var[Y] = \mathbb{E}[Y^2] - \mu_Y^2 = 2\theta^2 \cdot e^{-\frac{d}{\theta}} - \theta^2 \left(e^{-\frac{d}{\theta}}\right)^2$
$= \theta^2 \cdot e^{-\frac{d}{\theta}} \left(2 - e^{-\frac{d}{\theta}}\right)$
$= (10000)^2 \cdot e^{-3} \left(2 - e^{-3}\right) = 9709538$

$\Rightarrow \sigma_Y = 3116$ $\Rightarrow$ answer: $\frac{3116}{10000\, e^{-3}} = 6.2587$

OR:

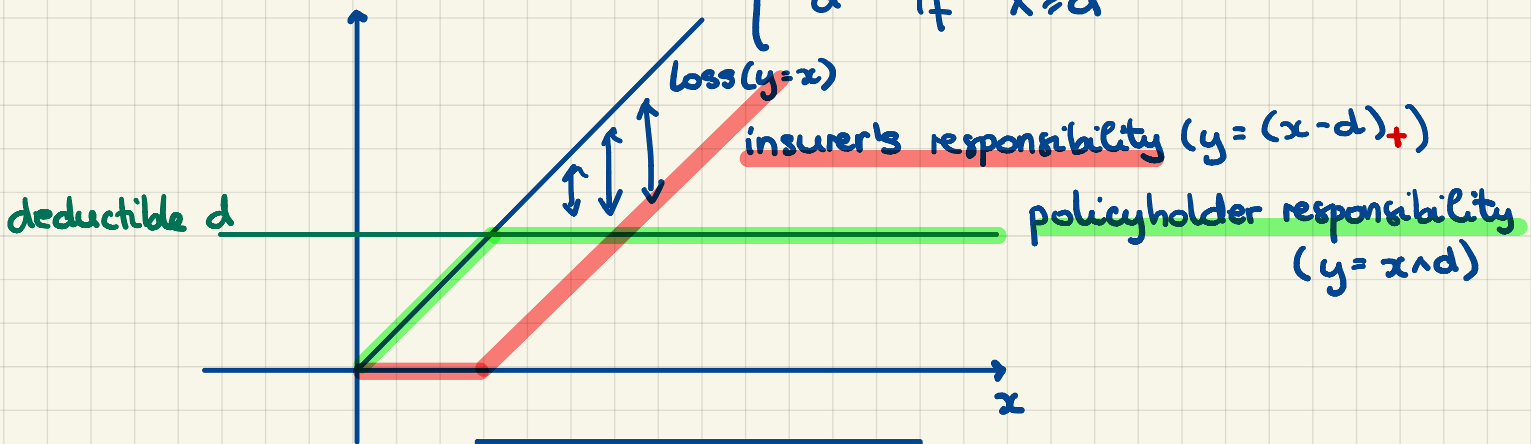
$$\frac{\sigma_Y}{\mu_Y} = \frac{\cancel{\theta} \cdot e^{-\frac{d}{2\theta}} \sqrt{2-e^{-\frac{d}{\theta}}}}{\cancel{\theta} \cdot e^{-\frac{d}{\theta}}} = e^{\frac{d}{2\theta}} \sqrt{2-e^{-\frac{d}{\theta}}}$$

$$= e^{1.5}\sqrt{2-e^{-3}} = \underline{6.2587} \quad \square$$

Def'n. The **limited loss (random) variable** is $\boxed{X \wedge d}$.

Note:

$$X \wedge d = \min(X, d) = \begin{cases} X & \text{if } X < d \\ d & \text{if } X \geq d \end{cases}$$



Algebraically: $\boxed{x \wedge d + (x-d)_+ = x}$

$\Rightarrow$ For X being our severity r.v., we get

$$X \wedge d + (X-d)_+ = X$$

Apply the expectation to this equality:

$$\mathbb{E}[X \wedge d] + \mathbb{E}[(X-d)_+] = \mathbb{E}[X]$$

$$\boxed{\begin{aligned} &\mathbb{E}[Y^L] = \mathbb{E}[X] - \mathbb{E}[X \wedge d] \\ &\mathbb{E}[Y^P] = \frac{\mathbb{E}[Y^L]}{S_X(d)} = \frac{\mathbb{E}[X] - \mathbb{E}[X \wedge d]}{S_X(d)} = e_X(d) \end{aligned}}$$