M339J: April 21st, 2023.

Problem. A compound Poisson claim dist'n has the parameter λ equal to 4 and individual claim amounts X distributed as:

$$p_X(3)=0.4 \quad \text{and} \quad p_X(9)=0.6$$

What is the expected cost of an aggregate stop-loss insurance subject to a deductible of 3?

→ : $S = X_1 + X_2 + \dots + X_N$

w/ $N \sim \text{Poisson}(\lambda = 4)$

$$\mathbb{E}[(S-3)_+] = \boxed{\mathbb{E}[S]} - \boxed{\mathbb{E}[S \wedge 3]}$$

$$\mathbb{E}[S] = \mathbb{E}[N] \cdot \mathbb{E}[X] = 4 \cdot (3 \cdot 0.4 + 9 \cdot 0.6) = 4 \cdot 6.6 = \underline{26.4}$$

$$S \wedge 3 \sim \begin{cases} 0 & \text{w/ probab. } p_N(0) = e^{-4} \\ 3 & \text{w/ probab. } 1 - p_N(0) = 1 - e^{-4} \end{cases}$$

$$\mathbb{E}[S \wedge 3] = 3(1-e^{-4}) = \underline{2.945}$$

$$\mathbb{E}[(S-3)_+] = 26.4 - 2.945 = 23.455$$

□

Problem. Consider a discrete r.v. X whose pmf has the following form:

-1	0	1
$1-3p$	$2p$	p

The parameter p is unknown.
You observe:

$$1, -1, 0, -1, 0, 0$$

What is the maximum likelihood estimate for the parameter p?

$\longrightarrow$: $L(p) = (1-3p)^2 \cdot (2p)^3 \cdot p \propto (1-3p)^2 \cdot p^4$

$\ell(p) = 2 \cdot \ln(1-3p) + 4 \cdot \ln(p)$

$\ell'(p) = 2 \cdot \frac{1}{1-3p}(-3) + \frac{4}{p} = 0$

$$\frac{6}{1-3p} = \frac{4}{p}$$

$$3p = 2-6p$$

$$\boxed{\hat{p}_{MLE} = \frac{2}{9}}$$

□

Problem. Consider this collective risk model:

- The claim count r.v. N is geometric w/ mean 4.
- The severity r.v. has the following pmf:

  $p_X(1) = 0.6$, $p_X(2) = 0.4$
- Assume independence.

There is an insurance which covers aggregate losses subject to a deductible of 2.

Find the expected value of aggregate payments.

$\longrightarrow$: $S = X_1 + X_2 + \cdots + X_N$ ✓, w/ $\boxed{N \sim g(\text{mean}=4)}$

$$\mathbb{E}[(S-2)_+] = \boxed{\mathbb{E}[S]} - \boxed{\mathbb{E}[S \wedge 2]} \checkmark$$

$$\mathbb{E}[S] = \mathbb{E}[N] \cdot \mathbb{E}[X] = 4\,(1 \cdot 0.6 + 2 \cdot 0.4) = 4 \cdot 1.4 = 5.6$$

$$S \wedge 2 \sim \begin{cases} 0 & \text{w/ prob. } p_N(0) = \frac{1}{5} \\ 1 & \text{w/ prob. } p_N(1) \cdot p_X(1) = \frac{4}{5} \cdot \frac{1}{5} \cdot 0.6 = \frac{12}{125} \\ 2 & \text{w/ prob. } 1 - \frac{1}{5} - \frac{12}{125} = \underline{88/125} \end{cases}$$

$$\mathbb{E}[S \wedge 2] = 1 \cdot \frac{12}{125} + 2 \cdot \frac{88}{125} = \frac{12 + 176}{125} = \frac{188}{125}$$

$$\mathbb{E}[(S-3)_+] = 5.6 - \frac{188}{125} = \underline{4.096} \quad \square$$