M339J: April 20th, 2022.

The Recursive Method.

N... frequency from the $(a,b,0)$ class: $\boxed{p_k = \left(a+\frac{b}{k}\right)\cdot p_{k-1}}$

X... severity on support $\{0,1,2,\dots,m\}$ (m could be ∞)

use $f_X(x)$ for the pmf of X for $x\in\{0,1,2,\dots,m\}$

$$\boxed{S = X_1 + X_2 + \dots + X_N}$$

There exists a recursion for $f_S(x)$, i.e., the pmf of S for $x \in \mathbb{N}_0$

- $f_S(0) = \mathbb{P}[S=0]$

$$= \boxed{\mathbb{P}[S=0 \mid N=0]}\cdot \mathbb{P}[N=0]$$
$$+ \mathbb{P}[S=0 \mid N=1]\cdot \mathbb{P}[N=1]$$
$$\vdots$$
$$+ \mathbb{P}[S=0 \mid N=k]\cdot \mathbb{P}[N=k] + \dots$$
$$= \boxed{1}\cdot p_N(0)$$
$$+ \boxed{f_X(0)}\cdot p_N(1)$$
$$\vdots$$
$$+ \boxed{(f_X(0))^k}\cdot p_N(k) + \dots$$
$$= \mathbb{E}\left[(f_X(0))^N\right] = P_N(f_X(0))$$

- $$f_S(x) = \boxed{\frac{1}{1-a\cdot f_X(0)}} \sum_{y=1}^{x\wedge m}\left(a+\frac{by}{x}\right) f_X(y)\cdot f_S(x-y)$$

for $x = 1,2,3,\dots$

A special Case: If $f_X(0)=0$, then

- $f_S(0) = \mathbb{P}[N=0] = p_N(0)$
- $$f_S(x) = \sum_{y=1}^{x \wedge m} \left(a + \frac{by}{x}\right) f_X(y) f_S(x-y)$$

In particular: Poisson $\Rightarrow$ $a=0$, $b=\lambda$

- $f_S(0) = e^{-\lambda}$
- $$f_S(x) = \frac{\lambda}{x} \sum_{y=1}^{x \wedge m} y \cdot f_X(y) f_S(x-y)$$

Geometric $\Rightarrow$ $a = \dfrac{\beta}{1+\beta}$, $b=0$

- $f_S(0) = \dfrac{1}{1+\beta}$
- $$f_S(x) = \frac{\beta}{1+\beta} \sum_{y=1}^{x \wedge m} f_X(y) f_S(x-y)$$

8. Annual aggregate losses for a dental policy follow the compound Poisson distribution with $\lambda = 3$. The distribution of individual losses is:

$f_X(0)=0$

Loss	Probability
1	0.4
2	0.3
3	0.2
4	0.1

Calculate the probability that aggregate losses (S) in one year do not exceed 3.

$\mathbb{P}[S \le 3] = ?$

$= \boxed{f_S(0)} + f_S(1) + f_S(2) + f_S(3) = e^{-3} + 1.2e^{-3} + 1.62e^{-3} + 1.968e^{-3}$

$= 0.28817$

(A) Less than 0.20

(B) At least 0.20, but less than 0.40

(C) At least 0.40, but less than 0.60

(D) At least 0.60, but less than 0.80

(E) At least 0.80

$f_X(0)=0$

we can use the "special case" formula

- $f_S(0) = e^{-3}$
- $f_S(1) = \frac{3}{1}\cdot 1 \cdot f_X(1) \cdot \boxed{f_S(1-1)} = 3(0.4)e^{-3} = \boxed{1.2e^{-3}}$
- $f_S(2) = \frac{3}{2}\left(1\cdot f_X(1)\cdot \boxed{f_S(2-1)} + 2\cdot f_X(2)\cdot \boxed{f_S(2-2)}\right)$

$= \frac{3}{2}\left(0.4\cdot 1.2e^{-3} + 2\cdot 0.3\cdot e^{-3}\right) = \boxed{1.62\, e^{-3}}$

- $f_S(3) = \frac{3}{3}\left(1\cdot f_X(1)\boxed{f_S(3-1)} + 2\cdot f_X(2)\cdot f_S(3-2) + 3\cdot f_X(3)\cdot f_S(3-3)\right)$

$= 0.4\cdot 1.62e^{-3} + 2\cdot 0.3\cdot 1.2e^{-3} + 3\cdot 0.2\cdot e^{-3} = 1.968\, e^{-3}$