18.2. In a study of 1,000 people with a particular illness, 200 died within one year of diagnosis. Calculate a 95% (linear) confidence interval for the one-year empirical survival function.

(A) (0.745, 0.855)

(B) (0.755, 0.845)

(C) (0.765, 0.835)

(D) (0.775, 0.825)

(E) (0.785, 0.815)

18.3. A cohort of 100 newborns is observed from birth. During the first year, 10 drop out of the study and one dies at time 1. Eight more drop out during the next six months, then, at time 1.5, three deaths occur.

Calculate $\hat{S}(1.5)$, the Nelson-Aalen ~~estimator~~ of the survival function, $S(1.5)$.

By def'n : $\boxed{\hat{S}(1.5) = e^{-\hat{H}(1.5)}}$

(A) 0.950

(B) 0.951

(C) 0.952

(D) 0.953

(E) 0.954

100 start and 10 drop out =>

90 left @ time 1

1 dies @ time 1 } => $\frac{s_1}{r_1} = \frac{1}{90}$

=> 89 left after time 1

w/ 8 dropping out in (1, 1.5)

=> 81 left @ time 1.5

3 deaths @ time 1.5 } => $\frac{s_2}{r_2} = \frac{3}{81}$

$\Rightarrow \hat{H}(1.5) = \frac{1}{90} + \frac{3}{81} = \underline{0.04815}$

$\Rightarrow \hat{S}(1.5) = e^{-0.04815} = \underline{0.953}$ $\square$

18.10 Initially, 50 lives are included in an observation of survival times following a specific medical treatment. You are given excerpted information from the study data in the table below.

j	$t_{(j)}$	Deaths at $t_{(j)}$	Exits (other than death) in $(t_{(j)}, t_{(j+1)}]$	Entrants in $(t_{(j)}, t_{(j+1)}]$
0			4	0
1	0.2	$\boxed{1}$	2	3
2	1.8	$\boxed{1}$	5	0
3	1.9	$\boxed{1}$	0	0
4	2.1	1	7	0

Calculate the Nelson-Aalen estimate of $S(2)$.

We need $\hat{H}(2)$.

(A) 0.910
(B) 0.916
(C) 0.922
(D) 0.928
(E) 0.934

$r_1 = 50 - 4 = 46 \Rightarrow \frac{1}{r_1} = \frac{1}{46}$

$\Rightarrow$ After time 0.2, we have $\boxed{46-1=45 \text{ left}}$

[Question on October 2022 FAM-L Exam]

Then, until time 1.8, 3 enter and 2 exit:

$r_2 = 45 + 3 - 2 = 46 \Rightarrow \frac{1}{r_2} = \frac{1}{46}$

$\Rightarrow 46 - 1 = 45$ left

Until time 1.9, 5 exit

$\Rightarrow r_3 = 45 - 5 = 40 \Rightarrow \frac{1}{r_3} = \frac{1}{40}$

$\Rightarrow \hat{H}(2) = \frac{1}{46} + \frac{1}{46} + \frac{1}{40} = \underline{0.068478}$

$\Rightarrow \hat{S}(2) = e^{-\hat{H}(2)} = \underline{0.9338}$ □

7.40 For a fully discrete whole life insurance of 1000 on (60), you are given:

i) Reserves are determined using a modified net premium reserve method
ii) The modified reserve at the end of year 2 is 0
iii) Valuation premiums in years 3 and later are level
iv) Mortality follows the Standard Ultimate Life Table
v) $i = 0.05$

Calculate the modified net premium reserve at the end of year 5.

(A) 58
(B) 69
(C) 79
(D) 90
(E) 99

[Question on October 2022 FAM-L Exam]

18.1. An insurer is modelling time to death of lives insured at age x using the Kaplan-Meier estimator. You are given the following information.

(i) There were 100 policies in force at time 0

(ii) There were no new policies entering the study

(iii) At time 10.0, immediately after a death, there were 50 policies remaining in force

(iv) The Kaplan-Meier estimate of the survival function for death at time 10 is $\boxed{\hat{S}(10.0) = 0.92}$ ✓

(v) The next death after time 10.0 occurred when there $\boxed{\text{was one death at time 10.8}}$

(vi) During the period from time 10.0 to time 10.8, a total of 10 policies terminated for reasons other than death

$r_j = 50 - 10 = 40$

Calculate $\hat{S}(10.8)$, the Kaplan-Meier estimate of the survival function $S(10.8)$.

$\rightarrow$:

contribution :

$\left(\frac{40-1}{40}\right)$

(A) 0.897
(B) 0.903
(C) 0.909
(D) 0.910
(E) 0.920

$\Rightarrow \hat{S}(10.8) = 0.92 \cdot \left(\frac{39}{40}\right) = \underline{0.897}$ $\square$

18.9 Initially, 80 lives are included in an observation of survival times following a specific medical treatment. You are given excerpted information from the study data in the table below.

j	$t_{(j)}$	Deaths at $t_{(j)}$	Exits (other than death) in $(t_{(j)}, t_{(j+1)}]$	Entrants in $(t_{(j)}, t_{(j+1)}]$
0			20	4
1	0.5	$\boxed{1}$	2	3
2	1.6	1	6	0
3	1.9	1	8	0
4	2.5	1	10	0

Calculate the Kaplan-Meier estimate of $S(2)$.

(A) 0.931

(B) 0.952

(C) 0.960

(D) 0.969

(E) 0.972

[Question on October 2022 FAM-L Exam]

- 80 to start − 20 exits + 4 entrants

$\Rightarrow r_1 = 64$ one death at time 0.5 $\Rightarrow \frac{63}{64}$

- 63 remain − 2 exits + 3 entrants

$\Rightarrow r_2 = 64$ one death @ time 1.6 $\Rightarrow \frac{63}{64}$

- 63 remain − 6 exits (+ 0 entrants)

$\Rightarrow r_3 = 57$ one death @ time 1.9 $\Rightarrow \frac{56}{57}$

$$\Rightarrow \hat{S}(2) = \left(\frac{63}{64}\right)^2 \cdot \frac{56}{57} = 0.952$$

196. You are given the following 20 bodily injury losses (before the deductible is applied):

Loss	Number of Losses	Deductible	Policy Limit
✓ 750	3	200	∞
✓ 200	3	0	10,000
✓ 300	4	0	20,000
✓ >10,000	6	0	10,000
400	4	300	∞

T : conditional pdf
U : pdf
U : pdf
RC : survival f'tion
T : conditional pdf

Past experience indicates that these losses follow a Pareto distribution with parameters α and $\theta = 10,000$.

Calculate the maximum likelihood estimate of α.

$$\longrightarrow : \; L(\alpha) = \boxed{\left(\frac{f_X(750)}{S_X(200)}\right)^3} \cdot \boxed{\left(f_X(200)\right)^3} \boxed{\left(f_X(300)\right)^4} \boxed{\left(S_X(10000)\right)^6} \left(\frac{f_X(400)}{S_X(300)}\right)^4$$

(A) Less than 2.0

(B) At least 2.0, but less than 3.0

(C) At least 3.0, but less than 4.0

(D) At least 4.0, but less than 5.0

(E) At least 5.0

$$L(\alpha) = \left(\frac{\alpha \cdot \cancel{\theta^\alpha}}{\boxed{(750+\theta)^{\alpha+1}}}\right)^3 \cdot \left(\frac{1}{\frac{\cancel{\theta^\alpha}}{\boxed{(200+\theta)^\alpha}}}\right)^{\boxed{3}}$$

$$\left(\frac{\alpha \, \boxed{\theta^\alpha}}{\boxed{(200+\theta)^{\alpha+1}}}\right)^3 \left(\frac{\alpha \, \boxed{\theta^\alpha}}{\boxed{(300+\theta)^{\alpha+1}}}\right)^4 \left(\frac{\boxed{\theta^\alpha}}{\boxed{(10000+\theta)^\alpha}}\right)^6$$

$$\cdot \left(\frac{\alpha \cdot \cancel{\theta^\alpha}}{(400+\theta)^{\alpha+1}}\right)^4 \cdot \left(\frac{1}{\frac{\cancel{\theta^\alpha}}{\boxed{(300+\theta)^\alpha}}}\right)^4$$

$$(10750)^{-3\alpha} \cdot \boxed{(10750)^{-3}} \text{ const}$$

$$L(\alpha) = \alpha^{14} \cdot \theta^{13\cdot\alpha} \cdot \boxed{(10750)^{-3(\alpha+1)}} \boxed{(10200)^{-3} (10300)^{-4}}$$

$$\cdot 20000^{-6\alpha} \quad (10400)^{-4(\alpha+1)} \qquad \text{const.}$$

$$L(\alpha) \propto \alpha^{14} \cdot \theta^{13\alpha} (10750)^{-3\alpha} \cdot (20000)^{-6\alpha} \cdot (10400)^{-4\alpha}$$

=> the loglikelihood:

$$\ell(\alpha) \text{ "=" } 14 \cdot \ln(\alpha) + 13 \cdot \alpha \cdot \ln(\theta) - 3\alpha \ln(10750) - 6\alpha \ln(20000)$$

$$- 4\alpha \cdot \ln(10400)$$

$$\Rightarrow \ell'(\alpha) = 14 \cdot \frac{1}{\alpha} + 13 \ln(10000) - 3 \ln(10750) - 6 \ln(20000) - 4 \ln(10400) = 0$$

$$\boxed{\hat{\alpha}_{MLE} = 3.089}$$

□