The University of Texas at Austin
In-Class Work 7

M339G Predictive Analytics

October 28, 2026

Bivariate Normal Distribution.

Problem 7.1. The extremely popular *Fit Fathers* exercise club team enrolled in a fitness regimen. Here is a summary of their pre- and post-regimen weights (in lbs).

	mean	standard deviation
PRE	220	20
POST	185	10

Assume that the data are modeled as bivariate normal with the correlation coefficient equal to 0.8. Of the fathers who weighed **exactly** 260 pounds before the fitness regimen, about what percentage weighed above average after the regimen?

Solution. Let (U, V) be the random pair which stands for the fathers' pre- and post-regimen weights in lbs. Let (X, Y) be the random pair which stands for their weights in standard units.

Conditioning on U being f exactly 260 is equivalent to conditioning on

$$X = \frac{260 - 220}{20} = 2$$

So, the probability that we are looking for is

$$\mathbb{P}[V > 185 \,|\, U = 260] = \mathbb{P}\left[\frac{V - 185}{10} > \frac{185 - 185}{10} \,|\, U = 260\right] = \mathbb{P}[Y > 0 \,|\, X = 2].$$

Recall that

$$Y \,|\, X = x \sim N\Big(mean = \rho x, sd = \sqrt{1 - \rho^2}\Big)$$

. So, the probability we are seeking equals (with $x = 2$)

$$\mathbb{P}\left[Z > \frac{0 - \rho x}{\sqrt{1 - \rho^2}}\right] = \mathbb{P}\left[Z > \frac{0 - 0.8(2)}{\sqrt{1 - 0.64}} \approx -2.67\right] \approx \mathbb{P}[Z \leq 2.67] = \Phi(2.67) = 0.9963.$$

Problem 7.2. *Source: Pitman's "Probability."*
Here is a summary of PSAT and SAT scores of a large group of students.

	mean	standard deviation
PSAT	1200	100
SAT	1300	90

Assume that the data are modeled as bivariate normal with the correlation coefficient equal to 0.6. Of the students who scored 1000 on the PSAT, about what percentage scored above average on the SAT?

Solution. Let (U, V) be the random pair which stands for the students' PSAT and SAT scores in real units. Let (X, Y) be the random pair which stands for the students' PSAT and SAT scores in standard units.

Conditioning on U being f exactly 1000 is equivalent to conditioning on

$$X = \frac{1000 - 1200}{100} = -2$$

So, the probability that we are looking for is

$$\mathbb{P}[Y > 0 \,|\, X = -2].$$

Recall that $Y \,|\, X = x \sim N(\rho x, 1 - \rho^2)$. So, the probability we are seeking equals (with $x = -2$)

$$\mathbb{P}\left[Z > \frac{0 - \rho x}{\sqrt{1 - \rho^2}}\right] = \mathbb{P}\left[Z > -\frac{x\rho}{\sqrt{1 - \rho^2}}\right]$$

With the given correlation coefficient of 0.6, our answer is

$$\Phi\left(\frac{-2(0.6)}{\sqrt{1 - 0.36}}\right) = \Phi(-1.5) = 0.0668072.$$