

The University of Texas at Austin
IN-CLASS WORK 2

M339G Predictive Analytics

September 16, 2026

FROM LINEAR ALGEBRA TOWARDS MULTIPLE LINEAR REGRESSION.

PROBLEM 2.1: Defining a hyperplane

In $\mathbb{R}^3$, a hyperplane is a 2-dimensional flat surface defined by an equation of the form

$$a_1x_1 + a_2x_2 + a_3x_3 = b$$

Write the equation of the hyperplane through the point $(1, 2, 3)$ with normal vector $\mathbf{a} = (2, -1, 4)$.

PROBLEM 2.2: Normal vector and orientation

For the hyperplane $3x_1 + 4x_2 - 5x_3 = 10$, identify the normal vector, and determine whether the point $(2, 1, 0)$ lies on the hyperplane, above it, or below it (i.e., compute $\mathbf{a} \cdot \mathbf{x} - b$ and interpret its sign).

PROBLEM 2.3: From a geometric hyperplane to a linear function

A hyperplane in $\mathbb{R}^n$ can be written as the graph of a function

$$x_n = \beta_0 + \beta_1 x_1 + \cdots + \beta_{n-1} x_{n-1}$$

by solving $\mathbf{a} \cdot \mathbf{x} = b$ for the last coordinate. Rewrite the hyperplane $2x_1 - 3x_2 + x_3 = 6$ (in $\mathbb{R}^3$) in the form $x_3 = \beta_0 + \beta_1 x_1 + \beta_2 x_2$, and identify $\beta_0, \beta_1, \beta_2$.

PROBLEM 2.4: Relabeling as a regression model

Suppose we now treat x_3 from the previous problem as a response variable y , and x_1, x_2 as predictors, but real data won't lie exactly on a hyperplane. Write the multiple linear regression model for one observation i , using coefficients $\beta_0, \beta_1, \beta_2$ and an error term ε_i , based on that same structural form.

PROBLEM 2.5: The regression hyperplane as a best fit

Given the model

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i$$

for n observations, explain in matrix form how the hyperplane's coefficients are chosen, and write the normal equations that determine $\hat{\beta}$.

