Bivariate Normal in Matrix Notation.

Consider a bivariate normal pair (U,V)

In 2D: we place the mean into a vector

$$\mu := \begin{pmatrix} \mu_U \\ \mu_V \end{pmatrix}$$

anything in $\mathbb{R}^2$

The variances/covariances are placed into a matrix:

$$\Sigma := \begin{bmatrix} \sigma_U^2 & \sigma_U \sigma_V \rho \\ \sigma_U \sigma_V \rho & \sigma_V^2 \end{bmatrix}$$

positive definite

In 1D:

$$f_W(w) = \frac{1}{\sigma_W \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{w-\mu_W}{\sigma_W}\right)^2}$$

Then, the joint density of (U,V) can be written as

$$f_{U,V}(u,v) = \frac{1}{2\pi} \cdot \frac{1}{(\det(\Sigma))^{1/2}} \exp\left(-\frac{1}{2}\begin{pmatrix} u-\mu_U \\ v-\mu_V \end{pmatrix}^T \Sigma^{-1} \begin{pmatrix} u-\mu_U \\ v-\mu_V \end{pmatrix}\right)$$

1×2 $\quad 2\times 2$ $\quad 2\times 1$

Task: What do you get for (X,Z) bivariate standard normal w/ correlation ρ?

Multivariate Normal Density.

Let $\mathbb{X} = (X_1, \ldots, X_p)^T$ be

$$\text{Normal}\left(\text{mean} = \mu = (\mu_1, \ldots, \mu_p)^T,\ \Sigma = \begin{bmatrix} \sigma_1^2 & \cdots & \text{Cov} \\ \text{Cov} & \ddots & \sigma_p^2 \end{bmatrix}\right)$$

w/ Σ positive definite

Then,

$$f_{\mathbb{X}}(\underbrace{x_1, \ldots, x_p}_{x}) = \frac{1}{(2\pi)^{p/2}} \cdot \frac{1}{(\det(\Sigma))^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

for all $x \in \mathbb{R}^p$