Confidence Intervals.

Let $(X_1, X_2, \dots, X_n)$ be a normal random sample, i.e.,

$\{X_1, \dots, X_n\}$ are independent

and

? known!

$X_i \sim \text{Normal}(\text{mean} = \mu, \text{sd} = \sigma)$

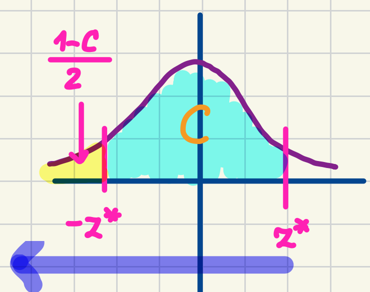
We know the sampling distribution of the sample mean

$$\underset{\frac{1}{n}(X_1 + \cdots + X_n)}{\underset{\parallel}{\bar{X}}} \sim \text{Normal}\left(\text{mean} = \mu, \text{sd} = \frac{\sigma}{\sqrt{n}}\right)$$

C ... confidence level.

$$\boxed{\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}} \sim N(0,1)$$

Pivotal Quantity.



$$\mathbb{P}\left[-z^* \le \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \le z^*\right] = C$$

$$\frac{1-C}{2} + C = \frac{1+C}{2}$$

$$\Phi(z^*) = \frac{1+C}{2}$$

$$\mathbb{P}\left[\bar{X} - z^* \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X} + z^* \cdot \frac{\sigma}{\sqrt{n}}\right] = C$$

$$z^* = \Phi^{-1}\left(\frac{1+C}{2}\right)$$

$= \text{qnorm}((1+C)/2)$

Confidence Interval

What if σ is not known?

Idea: Use the sample sd instead of σ!

$\Rightarrow$? $\boxed{\dfrac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} \sim t(df = n-1)}$

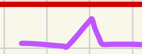
What if the sample is not normal?

Still, we're estimating μ.

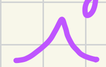
How do you triage?

eyeballing:
- histogram $\rightarrow$ symmetry and unimodality
- box plot $\rightarrow$ symmetry and outliers
- q-q plot $\rightarrow$ normality

need more:



triangular;



Laplace / double Exponential

- Shapiro-Wilk Test

What if normality fails?

$\rightarrow$ Sufficiently large data set $\overset{\boxed{CLT}}{\Longrightarrow}$ $\begin{cases} z\text{-procedures} \\ t\text{-procedures} \end{cases}$

$\rightarrow$ **BOOT STRAP**

Data Set: $x_1, \dots, x_n$

Draw with replacement

B... the # of resamplings

$$x_1, \dots, x_n \begin{cases} x_{11}, \dots, x_{1n} \rightarrow \bar{x}_1 \\ x_{21}, \dots, x_{2n} \rightarrow \bar{x}_2 \\ \vdots \\ x_{B1}, \dots, x_{Bn} \rightarrow \bar{x}_B \end{cases}$$

$\downarrow$

Confidence intervals.