The University of Texas at Austin
Homework Assignment 2

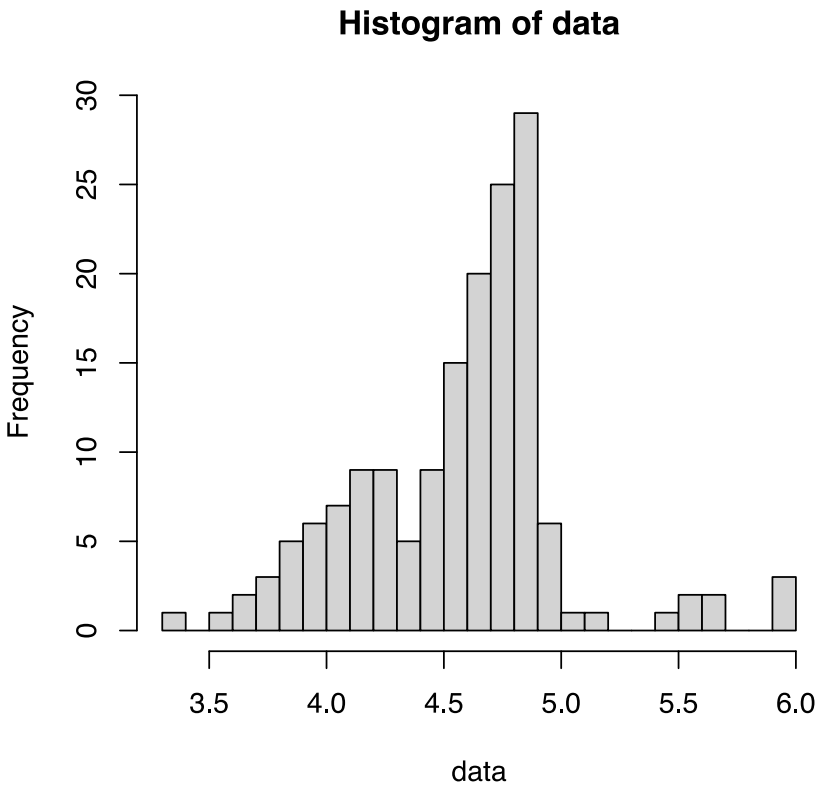
Predictive Analytics

September 07, 2026

Instructions: Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Bootstrap.

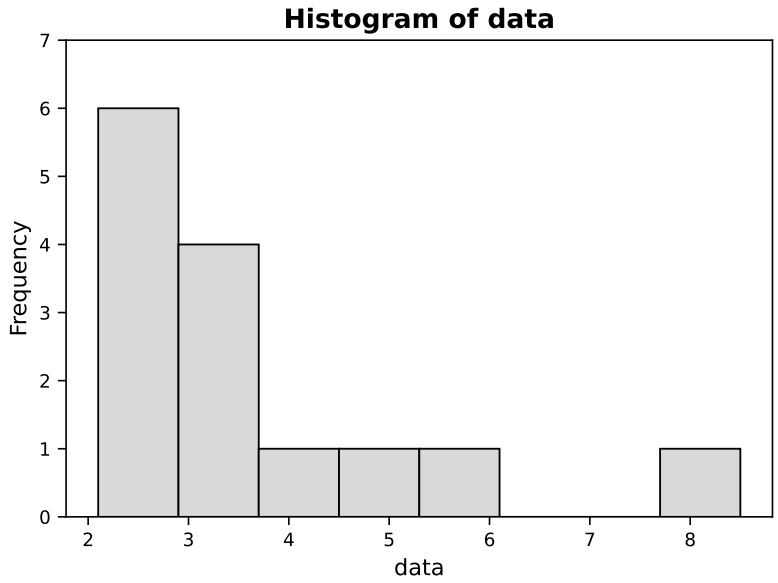
Problem 2.1: *(5 points) You have a sample of size 252 from a distribution that you know from past experience looks like this:*



Your task is to estimate its mean. Of the following, what procedure(s) would be acceptable in this case? Choose ***all*** *that apply.*

a. *A 95% bootstrap confidence interval using quantiles.*
b. *Using the 't.test' command in* ***R****.*
c. *A $2SE$ bootstrap confidence interval.*
d. *The standard z —procedure 95%-confidence interval.*
e. *None of the above.*

Problem 2.2: *(5 points) You have a sample of size* 14 *from a distribution that you know from past experience looks like this:*



Your task is to estimate its mean. Of the following, what procedure(s) would be acceptable in this case? Choose ***all*** *that apply.*

a. *A* 95% *bootstrap confidence interval using quantiles.*
b. *Using the 't.test' command in* ***R****.*
c. *A* $2SE$ *bootstrap confidence interval.*
d. *The standard* z *—procedure 95%-confidence interval.*
e. *None of the above.*

Problem 2.3: *(*15 *points) Consider a sample of size* n*. You intend to use bootstrap to estimate a quantity.*

i. *(*10 *points) Every time you resample, how many unique values from the original sample do you expect to see in the resample?*
ii. *(*5 *points) What is the limit of the proportion of unique values from the original sample in the resample as the sample size goes to infinity?*

Problem 2.4: *(*25 *points) Let* $X_1, ..., X_n$ *be an random sample from a continuous distribution, and consider its* n^{th} *order statistic* $X_{(n)}$*. As usual, we draw a bootstrap resample* $X_1^*, ..., X_n^*$ *by sampling* ***with replacement****. Let* $X_{(n)}^* = \max(X_1^*, ..., X_n^*)$ *be the* n^{th} *order statistic of the .*

i. *(*10 *points) Calculate that* $P\left(X_{(n)}^* = X_{(n)}\right)$*.*
ii. *(*5 *points) What does this probability converge to as* $n \to \infty$*?*
iii. *(*10 *points) Contrast the utility of bootstrap for the sample mean and the* n^{th} *order statistics. More precisely, explain briefly why the above shows that the bootstrap does* ***not*** *consistently estimate the sampling distribution of the sample maximum, even though it works beautifully for the sample mean.*