The University of Texas at Austin
In-Class Work 2

M339D Introduction to Financial Mathematics

September 03, 2026

Problem 2.1: *Write down the **definition** of the **effective interest rate** for the time period $[t_1, t_2]$ in terms of the accumulation fuction $a(\cdot)$.*

Solution 2.2:

$$i_{[t_1,t_2]} = \frac{a(t_2) - a(t_1)}{a(t_1)}$$

Problem 2.3: *Find the total amount of interest that would be paid on a \$1,000 loan over a $10-$year period, if the effective interest rate is 0.09 per annum and the entire loan plus entire accumulated interest is paid as one lump sum at the end of the loan term.*

Solution 2.4: *Using compound interest, the accumulated value at the end of the 10 years is*

$$1000 \cdot 1.09^{10} \approx 2367.36.$$

The total amount of interest is

$$2367.36 - 1000 = 1367.36.$$

Problem 2.5: Source: Exam FM/2, May 2005, Problem #7.
Mike receives cash flows of 100 today, 200 in one year, and 100 in two years. The net present value of these cash flows is 364.46 at an annual effective interest rate i. Calculate i.

a. *About* 10%
b. *About* 11%
c. *About* 12%
d. *About* 13%
e. *None of the above*

Solution 2.6: ***(a)***
The condition on the NPV is

$$364.46 = 100 + 200v + 100v^2$$

with $v = 1/(1+i)$. So, v must satisfy the following quadratic:

$$100v^2 + 200v - 264.46 = 0.$$

Hence,

$$v = \frac{-200 \pm \sqrt{40000 + 105784}}{200} \approx 0.909084.$$

Finally,

$$i = \frac{1}{v} - 1 \approx 0.10.$$

PROBLEM 2.7: *Write down the **definition** of the (time-varying) **force of interest** in terms of the accumulation fuction $a(\cdot)$.*

SOLUTION 2.8:

$$\delta_t = \frac{a'(t)}{a(t)} = \frac{d}{dt}[\ln(a(t))]$$

DEFINITION 2.9: *In the context of financial markets, the constant force of interest is usually referred to as the **continuously compounded, risk-free interest rate** and denoted by r.*

PROBLEM 2.10: *Let the accumulation function be given by*

$$a(t) = (1 + 0.05)^{3t}(1 + 0.02)^{t/2}.$$

Then, we can say the following about the continuously compounded, risk-free interest rate r associated with the above accumulation function:

a. $r = 0.07$
b. $r = \ln(1.05) + \ln(1.02)$
c. $r = 3\ln(1.05) + 0.5\ln(1.02)$
d. The continuously compounded, risk-free interest rate is not constant.
e. None of the above

SOLUTION 2.11: ***(c)***

$$r = \frac{d}{dt}\ln(a(t)) = \frac{d}{dt}\ln\left[1.05^{3t}1.02^{t/2}\right] = 3\ln(1.05) + 0.5\ln(1.02).$$

PROBLEM 2.12: *Let r be the continuously compounded, risk-free interest rate. What is the expression for the accumulation function $a(\cdot)$ in terms of r?*

SOLUTION 2.13:

$$a(t) = e^{rt}$$

PROBLEM 2.14: *Roger deposits \$100 into an account at time 0. He does not make any withdrawals or deposits and the account earns at a continuously compounded, risk-free interest rate r. After 15 years and 6 months, the balance in his account equals \$133. Then,*

a. $0 \leq r < 0.0150$
b. $0.0150 \leq r < 0.0250$
c. $0.0250 \leq r < 0.0550$
d. $0.0550 \leq r < 0.0650$
e. *None of the above*

SOLUTION 2.15: ***(b)***
The unknown continuously compounded, risk-free interest rate r must satisfy

$$133 = 100e^{15.5r}.$$

So,

$$r = \ln(1.33)/15.5 \approx 0.018.$$