The University of Texas at Austin
Homework Assignment 15

M339D Introduction to Financial Mathematics

October 26, 2026

Instructions: Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Multiple binomial periods.

Problem 15.1. Consider a non-dividend-paying stock whose current price is \$90 per share. The stock has the volatility equal to 0.20.

Let the continuously compounded, risk-free interest rate be equal to 0.03.

You model the evolution of this stock over the next year with a three-period **forward** binomial tree.

What is the price of a \$95-strike, one-year put on the above stock consistent with this model?

Solution. In our usual notation, in the forward binomial tree, the risk-neutral probability is

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.20\sqrt{1/3}}} = 0.4711645$$

The *up* and *down* factors are

$$u = e^{rh+\sigma\sqrt{h}} = e^{0.03(1/3)+0.20\sqrt{1/3}} = 1.133681,$$
$$d = e^{rh-\sigma\sqrt{h}} = e^{0.03(1/3)-0.20\sqrt{1/3}} = 0.8999014.$$

Hence, the possible stock prices at the end of the three periods are

$$S_{uuu} = S(0)u^3 = 131.134, \quad S_{uud} = S(0)u^2d = 104.0925,$$
$$S_{udd} = S(0)ud^2 = 82.62726, \quad S_{ddd} = S(0)d^3 = 65.58844.$$

So, the put option is in the money only in the two bottom nodes where the payoff equals

$$V_{udd} = (K - S_{udd})_+ = 12.37274 \quad \text{and} \quad V_{ddd} = (K - S_{ddd})_+ = 29.41156.$$

By the risk-neutral pricing formula, we have that

$$V_P(0) = e^{-0.03(1)}\left[12.37274 \times 3(0.4711645)(1-0.4711645)^2 + 29.41156 \times (1-0.4711645)^3\right] = 8.967831.$$

Problem 15.2. Let the be 0.10. Let the initial price of a non-dividend-paying stock be \$100 per share. You use a five-period binomial tree to model the stock price over the next year. Let $u = 1.04$ and $d = 0.96$.

What is the price of a one-year, at-the-money European call option on the above stock?

Solution. The risk-neutral probability is

$$p^* = \frac{e^{0.10(1/5)} - 0.96}{1.04 - 0.96} = 0.7525.$$

The relevant final possible stock prices in the binomial tree are

$$\begin{aligned} s_{5,5} &= S(0)u^5 = 100(1.04)^5 = 121.67, \\ s_{5,4} &= S(0)u^4 d = 100(1.04)^4(0.96) = 112.31, \\ s_{5,3} &= S(0)u^3 d^2 = 100(1.04)^3(0.96)^2 = 103.67. \end{aligned}$$

The remaining terminal nodes are f out of the money.

The possible payoffs are

$$v_{5,5} = 21.67, \quad v_{5,4} = 12.31, \quad v_{5,3} = 3.67.$$

So, the price of our call is

$$V_C(0) = e^{-0.10}\left[21.67(p^*)^5 + 12.31(5)(p^*)^4(1-p^*) + 3.67(10)(p^*)^3(1-p^*)^2\right] = 10.0176.$$