The University of Texas at Austin
Problem Set 14

M339D Introduction to Financial Mathematics

October 23, 2026

Instructions: Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Two periods.

One period [review].

Problem 14.1. *Source: SoA Exam FAM Sample Question #108.*
A put option has strike price $K = 70$ and matures in one year.

You are given that:

- the current price of the underlying stock is $S(0) = 75$;
- the stock price in one year will be either 100 or 60;
- the true probability of an upward price movement is 40%;
- the continuously compounded risk-free interest rate is $r = 0.03$.

Calculate the price of the option.

(A) 5.51
(B) 5.82
(C) 6.53
(D) 7.92
(E) 8.27

Solution. The true probability of an upward movement is irrelevant for pricing. The risk-neutral probability is

$$p^* = \frac{75e^{0.03} - 60}{100 - 60} = 0.4321.$$

The payoffs at the end of the period are

$$V_u = \max(0, 70 - 100) = 0, \quad V_d = \max(0, 70 - 60) = 10.$$

So, taking the expected value at time 0 of the payoff with respect to the risk-neutral probability, we get that the price of this put should be

$$e^{-0.03}[V_u \times p^* + V_d \times (1 - p^*)] = e^{-0.03} \times 10 \times 0.5679 = 5.51.$$

The answer is **(A)**.

Two periods.

Problem 14.2. For a two-period binomial model, you are given that:

- each period is one year;
- the current price of a non-dividend-paying stock S is $S(0) = \$100$;
- in each period, the stock price either goes up by 4% or goes down by 2%
- the continuously compounded risk-free interest rate is $r = 0.03$.

Consider a European put option on the above stock with strike price $K = 100$ and exercise date in two years.

Find the price of this put option.

Solution. In the standard notation for the binomial model, $u = 1.04$ and $d = 0.98$. The risk-neutral probability is

$$p^* = \frac{e^{0.03} - 0.98}{1.04 - 0.98} = 0.8409.$$

When one constructs the two-period binomial tree, one gets

$$S_u = 104, S_d = 98,$$
$$S_{uu} = 108.16, S_{ud} = S_{du} = 101.92, S_{dd} = 96.04.$$

So, the payoffs at the end of the second period are

$$V_{uu} = 0, \quad V_{ud} = 0, \quad V_{dd} = 3.96.$$

Taking the expected value at time 0 of the payoff with respect to the risk-neutral probability, we get that the price of this put should be

$$\begin{aligned} e^{-0.03\times 2}\left[V_{uu} \times (p^*)^2 + V_{ud} \times 2p^*(1-p^*) + V_{dd} \times (1-p^*)^2\right] \\ = e^{-0.06}\left[3.96 \times 0.1591^2\right] \\ = 0.0944. \end{aligned}$$

Forward binomial tree.

Problem 14.3. You are using a two-period forward binomial tree to model the price of a non-dividend-paying stock. You are given that:

- the current stock price is $S(0) = 100$;
- the volatility of the stock is $\sigma = 0.30$;
- the continuously compounded risk-free interest rate is $r = 0.05$;
- each period is six months long.

Consider a European call option on the above stock with strike price $K = 105$ and exercise date in one year.

Find the price of this call option.

Solution. The length of each period is $h = 0.5$. In the forward binomial tree,

$$u = e^{rh+\sigma\sqrt{h}} = e^{0.05\times 0.5+0.30\sqrt{0.5}} = 1.2676,$$
$$d = e^{rh-\sigma\sqrt{h}} = e^{0.05\times 0.5-0.30\sqrt{0.5}} = 0.8293.$$

The risk-neutral probability is

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.30\sqrt{0.5}}} = 0.4472.$$

When one constructs the two-period binomial tree, one gets

$$S_{uu} = 160.68, \quad S_{ud} = 105.13, \quad S_{dd} = 68.78.$$

So, the payoffs at the end of the second period are

$$V_{uu} = 55.68, \quad V_{ud} = 0.13, \quad V_{dd} = 0.$$

Taking the expected value at time 0 of the payoff with respect to the risk-neutral probability, we get that the price of this call should be

$$\begin{aligned} e^{-0.05\times 1}[V_{uu}\times(p^*)^2 + V_{ud}\times 2p^*(1-p^*) + V_{dd}\times(1-p^*)^2] \\ = e^{-0.05}[55.68\times 0.4472^2 + 0.13\times 2\times 0.4472\times 0.5528] \\ = 10.65. \end{aligned}$$

Special circumstances.

Problem 14.4. *Source: SoA Exam FAM Sample Question #103.*
The price movements of a non-dividend-paying stock are modeled by a binomial tree with one-year periods.

You are given that:

- the current price of the stock is $S(0) = 100$;
- $u = 1.24$, with u as in the standard notation for the binomial model;
- the risk-neutral probability of an up move is $p^* = 0.406$;
- the continuously compounded risk-free interest rate is $r = 0.02$.

The price of a two-year K-strike European put option on the stock is 25.05.

Calculate K.

(A) 123.50
(B) 124.00
(C) 124.50
(D) 125.00
(E) 125.50

Solution. From the formula for the risk-neutral probability,

$$p^* = \frac{e^{0.02} - d}{u - d} \quad \Rightarrow \quad d = \frac{e^{0.02} - up^*}{1 - p^*} = \frac{e^{0.02} - 1.24 \times 0.406}{0.594} = 0.87.$$

When one constructs the two-period binomial tree, one gets

$$S_{uu} = 153.76, \quad S_{ud} = 107.88, \quad S_{dd} = 75.69.$$

Since the put price is large relative to the stock price, we expect $107.88 < K < 153.76$. Then, the payoffs at the end of the second period are

$$V_{uu} = 0, \quad V_{ud} = K - 107.88, \quad V_{dd} = K - 75.69.$$

Taking the expected value at time 0 of the payoff with respect to the risk-neutral probability, we get

$$\begin{aligned} 25.05 &= e^{-0.02\times 2}[V_{ud} \times 2p^*(1-p^*) + V_{dd} \times (1-p^*)^2] \\ &= e^{-0.04}[(K - 107.88) \times 0.482328 + (K - 75.69) \times 0.352836]. \end{aligned}$$

So,

$$0.835164K - 78.7398 = 26.0723 \quad \Rightarrow \quad K = 125.50.$$

Since $107.88 < 125.50 < 153.76$, our assumption holds. The answer is **(E)**.

Problem 14.5. For a two-period binomial model, you are given that:

- each period is one year;
- the current price of a non-dividend-paying stock S is $S(0) = \$20$;
- $u = 1.2$, with u as in the standard notation for the binomial model;
- $d = 0.8$, with d as in the standard notation for the binomial model;
- the continuously compounded risk-free interest rate is $r = 0.04$.

Consider a **special** call option which pays the excess above the strike price $K = 23$ (if any!) at the end of **every** binomial period.

Find the price of this option.

Solution. The risk-neutral probability is

$$p^* = \frac{e^{0.04} - 0.8}{1.2 - 0.8} = 0.6020.$$

When one constructs the two-period binomial tree, one gets

$$S_u = 24, S_d = 16,$$
$$S_{uu} = 28.80, S_{ud} = S_{dd} = 19.2, S_{dd} = 12.8.$$

So, the payoffs at the end of the first period are

$$V_u = 1, V_d = 0.$$

The payoffs at the end of the second period are

$$V_{uu} = 5.80, \quad V_{ud} = 0, \quad V_{dd} = 0.$$

So, taking the expected value at time 0 of the payoff with respect to the risk-neutral probability, we get that the price of this call should be

$$\begin{aligned} e^{-0.04} \times V_u \times p^* + e^{-0.04\times 2}[V_{uu} \times (p^*)^2 + V_{ud} \times 2p^*(1-p^*)] \\ = e^{-0.04} \times 1 \times 0.6020 + e^{-0.08}[5.8 \times 0.6020^2] \\ = 2.51893. \end{aligned}$$