The University of Texas at Austin
In-Class Work 12

M339D Introduction to Financial Mathematics

October 21, 2026

Replicating Portfolios.

Problem 12.1. Let the be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a stock whose current price is \$80 per share. In the model, it is assumed that the stock price can either go up by \$5 or down by \$4.

You use the binomial tree to construct a replicating portfolio for a 78 —strike call option on the above stock. What is the stock investment in the replicating portfolio?

Solution. The two possible stock prices are $S_u = 85$ and $S_d = 76$. So, the possible payoffs of the call are $V_u = 8$ and $V_d = 0$. The Δ of the call, thus, equals

$$\Delta = \frac{V_u - V_d}{S_u - S_d} = \frac{8-0}{85-76} = \frac{8}{9}.$$

Problem 12.2. Let the be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a non-dividend-paying stock whose current price is \$50 per share. In the model, it is assumed that the stock price can either go up by 5% or down by 10%.

You use the binomial tree to construct a replicating portfolio for a 45 —strike call on the above stock. What is the risk-free investment in the replicating portfolio?

Solution. The two possible stock prices are $S_u = 52.5$ and $S_d = 45$. So, the possible payoffs of the call are $V_u = 7.5$ and $V_d = 0$. The risk-free investment B in the replicating portoflio of the call bull spread, thus, equals

$$B = e^{-rh}\frac{uV_d - dV_u}{u-d} = e^{-0.04}\frac{1.05(0) - 0.9(7.5)}{1.05 - 0.9} = -43.2355.$$

Problem 12.3. Consider a non-dividend paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously compounded, risk-free rate is 0.06.

A **straddle** consists of a long call and a long otherwise identical put. Consider a \$100-strike, one-year European straddle on the above stock. What is the straddle's price consistent with the above stock-price model?

a. About \$10
b. About \$10.83
c. About \$15.45
d. About \$20.84
e. None of the above.

Solution. (d)
The risk-neutral probability of an up movement is

$$p^* = \frac{95e^{0.06} - 75}{120 - 75} = 0.575.$$

So, the price of our straddle is

$$V(0) = e^{-0.06}[0.575 \times (120 - 100) + (1 - 0.575) * (100 - 75)] = 20.8366.$$