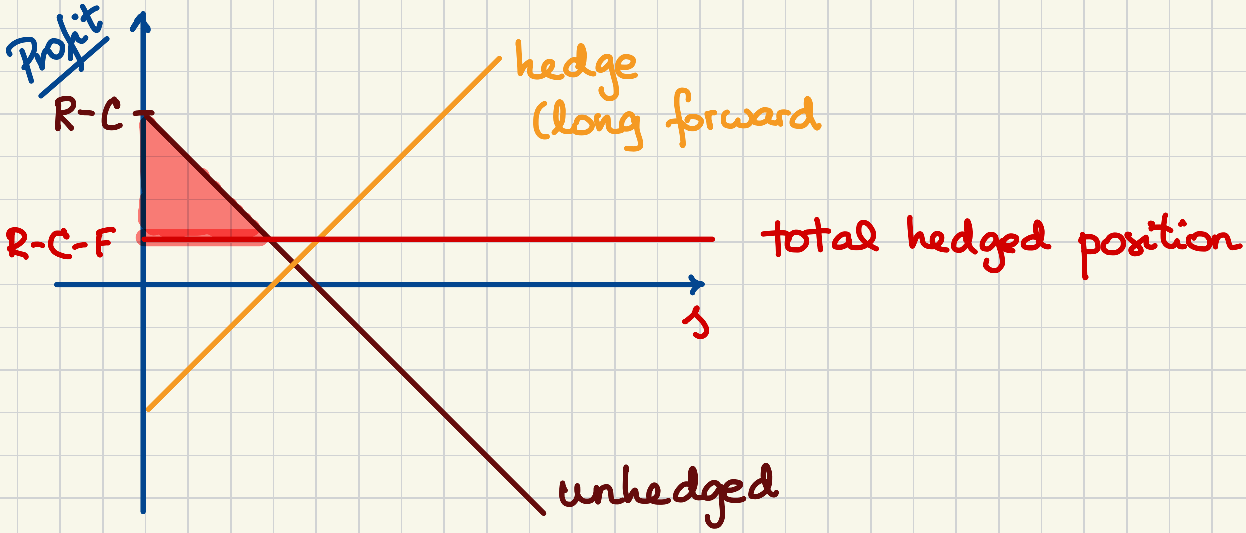
Inspiration.

Buyer/User of Goods

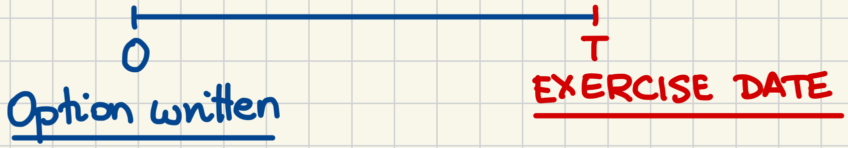


European **Call** **Options.**

European: The contract can only be exercised @ time T, i.e., its transaction can only take place on the exercise date.

Call: Usually, the right to buy the underlying asset for a predetermined price.

Options: Usually, the option's owner has the right, but not an obligation to exercise the option.



At time 0:
- The writer of the option writes/shorts the call.
- The buyer of the call is said to long the call. They are referred to as the option's owner.
- The agreement:
  - the underlying asset: $S(t), t \geq 0$
  - the exercise date: T
  - K... the strike/exercise price
- $\boxed{V_C(0)}$ is paid to the writer by the buyer!

At time T :
- The call's owner has a right but not an obligation to buy one unit of the underlying asset for the strike price K.
- The call's writer is obligated to do what the owner decides.

Payoff = ?

We focus on the payoff of the long call, i.e., the payoff for the call's owner.

The call owner's rationale for whether to exercise is to "maximize money in".

The criterion for exercise

IF $S(T) \geq K$, then EXERCISE. $\Rightarrow$ Payoff $= S(T) - K$

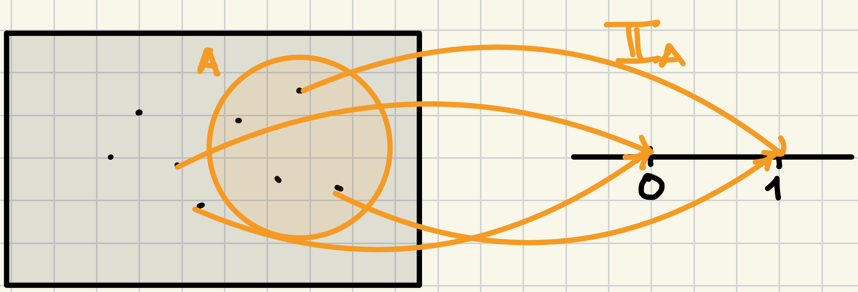
IF $S(T) < K$, then DO NOT EXERCISE. $\Rightarrow$ Payoff $= 0$

We introduce :

$V_C(T)$ ... the r.v. denoting the payoff of a long call

$$\Rightarrow \boxed{V_C(T) = \begin{cases} S(T) - K & \text{IF } S(T) \geq K \\ 0 & \text{IF } S(T) < K \end{cases}}$$

Indicator Random Variable.



Ω ... outcome space

$\omega \in \Omega$ ... elementary outcomes

A ... any "nice" subset of Ω aka an EVENT

We define: $\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{iff } \omega \in A \\ 0 & \text{iff } \omega \notin A \end{cases}$

$$\mathbb{I}_A = \begin{cases} 1 & \text{iff } A \text{ happened} \\ 0 & \text{iff } A \text{ did not happen} \end{cases}$$

$\Rightarrow$ $V_c(T) = (S(T)-K)\cdot \mathbb{I}_{\boxed{[S(T)\geq K]}}$

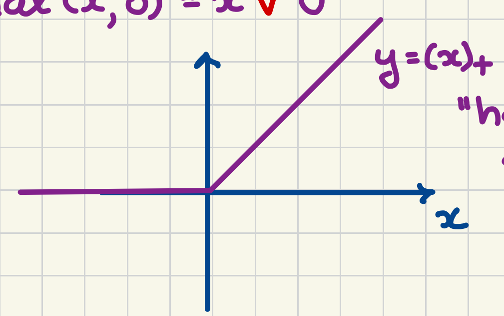
MAXIMUM OPERATOR

Also: $\boxed{V_c(T) = \max(S(T)-K, 0) = (S(T)-K) \vee 0}$

Note: $S(T)-K \geq 0 \iff S(T)\geq K$

Introduce: The Positive Part Function.

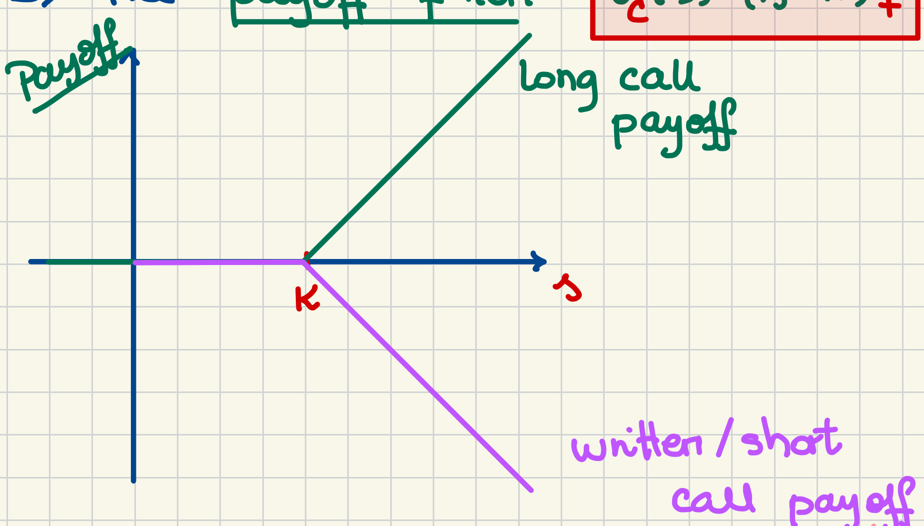
$$x \longmapsto (x)_+ := \max(x, 0) = x \vee 0$$



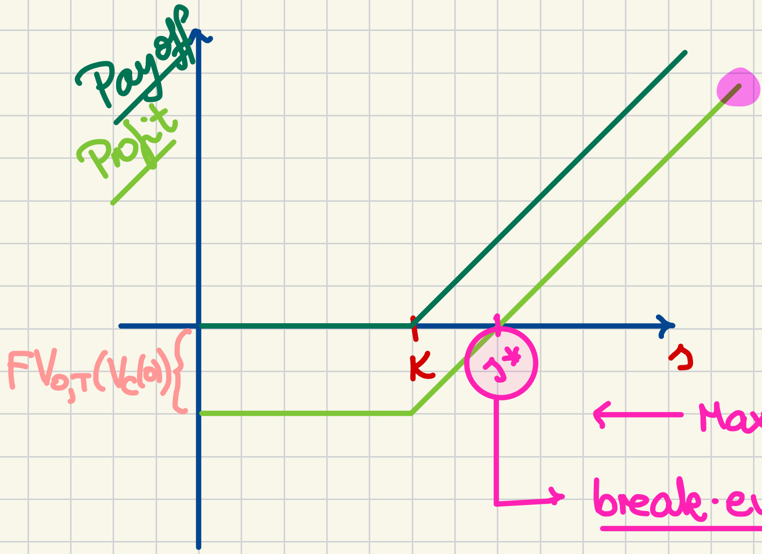
"hockey stick" f'tion

$\Rightarrow$ $\boxed{V_c(T) = (S(T)-K)_+}$

$\Rightarrow$ the payoff f'tion: $\boxed{v_c(s) = (s-K)_+}$



Never positive and sometimes strictly negative $\Rightarrow \exists$ premium $V_c(0) > 0$



Unlimited Growth Potential

$\boxed{\text{Long Position w.r.t. the Underlying}}$

← Maximum Loss

break-even: $\boxed{s^* = K + FV_{0,T}(V_c(0))}$

Problem 5.4. For what values of the final asset price is the profit of a long forward contract with the forward price $F = 100$ and delivery date T in one year smaller than the profit of a long call on the same underlying asset with the strike price $K = 100$ and the exercise date T. Assume that the call's premium equals \$10 and that the annual effective interest rate equals 10%.

Express your asnwer as an interval.