University of Texas at Austin

Problem Set 3

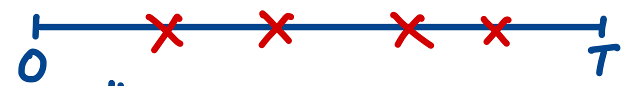
Payoff. Profit.

3.1. Static portfolios.

Step #1. Remember the **bottom-line approach** from *theory of interest*. Decide who your **protagonist** is!

Step #2. Set up the **timeline** (on paper or mentally):

static, i.e., no intermediate cashflows.



"today"

Cashflow from the perspective of our agent.

!!

Initial Cost

TIME HORIZON

Cashflow/Wealth from the perspective of our agent.

!!

Payoff

$$\boxed{\text{Profit} := \text{Payoff} - FV_{0,T}(\text{Initial Cost})}$$

This is how we will talk about **profit**:

- If Profit>0, then we call it a gain.
- If Profit<0, then we call it a loss.
- If Profit$=0$, then we say that we break even.

3.2. **Riskless assets.**

Example 3.1. Investing in a zero-coupon bond

$+C \ldots$ redemption amount

0 bond bought

T MATURITY DATE

$\boxed{r \ldots ccrfir}$ ✓

Initial Cost: Ce^{-rT}

Payoff: C

$$\text{Profit} = \text{Payoff} - FV_{0,T}(\text{Initial Cost})$$
$$= C - \underbrace{FV_{0,T}(Ce^{-rT})}_{C} = 0$$

□

Example 3.2. Taking a loan

$L \ldots$ loan amount

0 loan taken

T loan repaid

Initial Cost: $-L$

Payoff: $-Le^{rT}$

$$\text{Profit} = -Le^{rT} + FV_{0,T}(+L) = 0$$

3.3. **Risky assets.**

Example 3.3. Outright purchase of a stock

$S(t), t \geq 0 \ldots$ time-t stock price

0 — stock bought

T — stock sold

Initial Cost: $S(0)$

Payoff: $S(T)$ ... a random variable

Profit $= S(T) - FV_{0,T}(S(0))$

$= S(T) - S(0)e^{rT}$

Problem 3.1. Let the current price of a non-dividend-paying stock be \$40. The continuously compounded, risk-free interest rate is 0.04. You model the distribution of the time−1 price of the above stock as follows:

$$S(1) \sim \begin{cases} 45, & \text{with probability } 1/4, \\ 42, & \text{with probability } 1/2, \\ 38, & \text{with probability } 1/4. \end{cases}$$

What is your expected profit under the above model, if you invest in one share of stock at time−0 and liquidate your investment at time−1?

$\rightarrow$: $E[\text{Profit} = S(1) - FV_{0,1}(S(0))]$

$= E(S(1)) - E(FV_{0,1} S(0))$

$= 45 \cdot \frac{1}{4} + 42 \cdot \frac{1}{2} + 38 \cdot \frac{1}{4} - 40e^{.04}$

$= 41.75 - 40e^{0.04}$

$= .1176$

Goal. To study the payoff and the profit as **functions** of the **final asset price**.
Introduce. $s \dots$ an independent **argument** taking values in $[0, \infty)$ which will stand for the **final asset price**, i.e., it will be a "placeholder" for the random variable $S(T)$

Now, we can define the PAYOFF FUNCTION which describes the dependence of the payoff amount on the independent argument s.

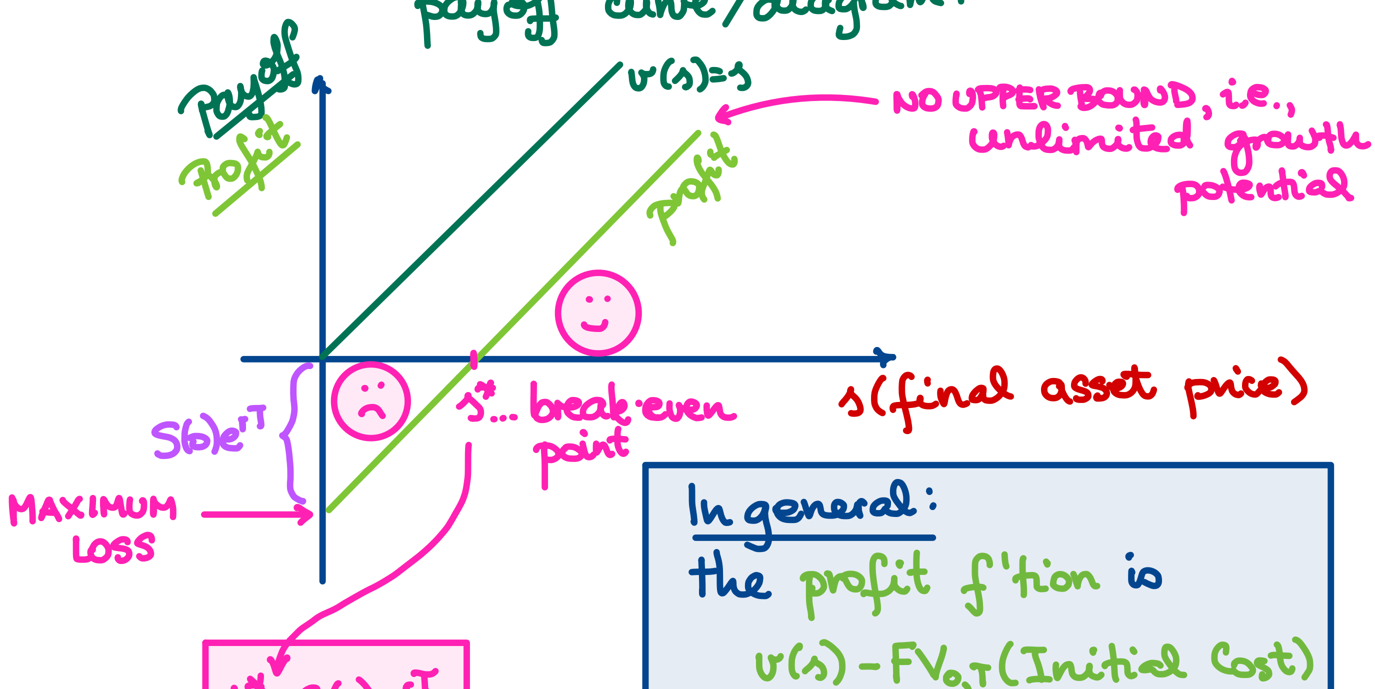
Notation: $v \dots$ payoff f'tion.

$v : [0, \infty) \longrightarrow \mathbb{R}$

$v(s) \dots$ the agent's payoff if the final asset price equals s

Example. For the outright purchase: $\boxed{v(s) = s}$ identity f'tion

When we plot the payoff f'tion, we get the payoff curve/diagram.



$\boxed{s^* = S(0)e^{rT}}$

In general: the profit f'tion is

$v(s) - FV_{0,T}(\text{Initial Cost})$

The payoff/profit f'tions are increasing.

Def'n. A function $f: \mathbb{R} \to \mathbb{R}$ is increasing if for all $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$

not necessarily strictly

Terminology:

If the payoff/profit is increasing (not necessarily strictly) as a function of the final asset price s, we say that the portfolio is

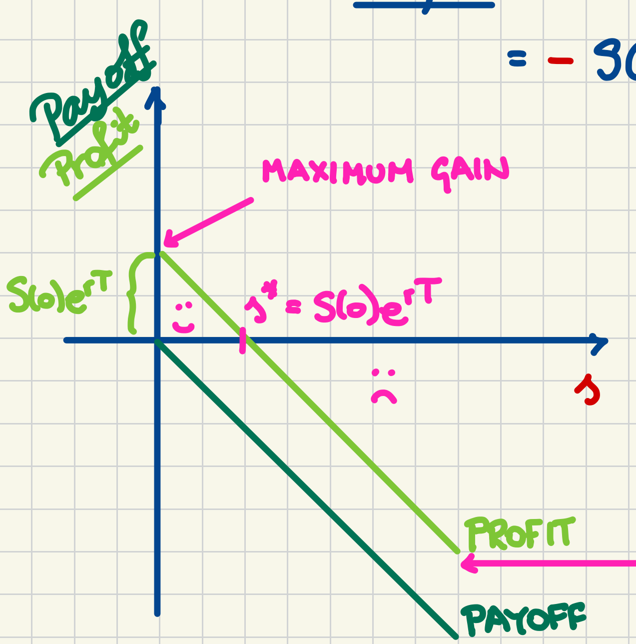
LONG w/ respect to the underlying asset.

Example. Short Sales.

Initial Cost: $-S(0)$

Payoff: $-S(T)$ $\Rightarrow$ payoff f'tion: $\boxed{v(s) = -s}$

$$\text{Profit} = -S(T) + FV_{0,T}(+S(0))$$
$$= -S(T) + S(0)e^{rT} \Rightarrow \boxed{\text{profit f'tion: } -s + S(0)e^{rT}}$$



The payoff/profit is decreasing, i.e., the short sale is short with respect to the underlying.

unlimited loss potential