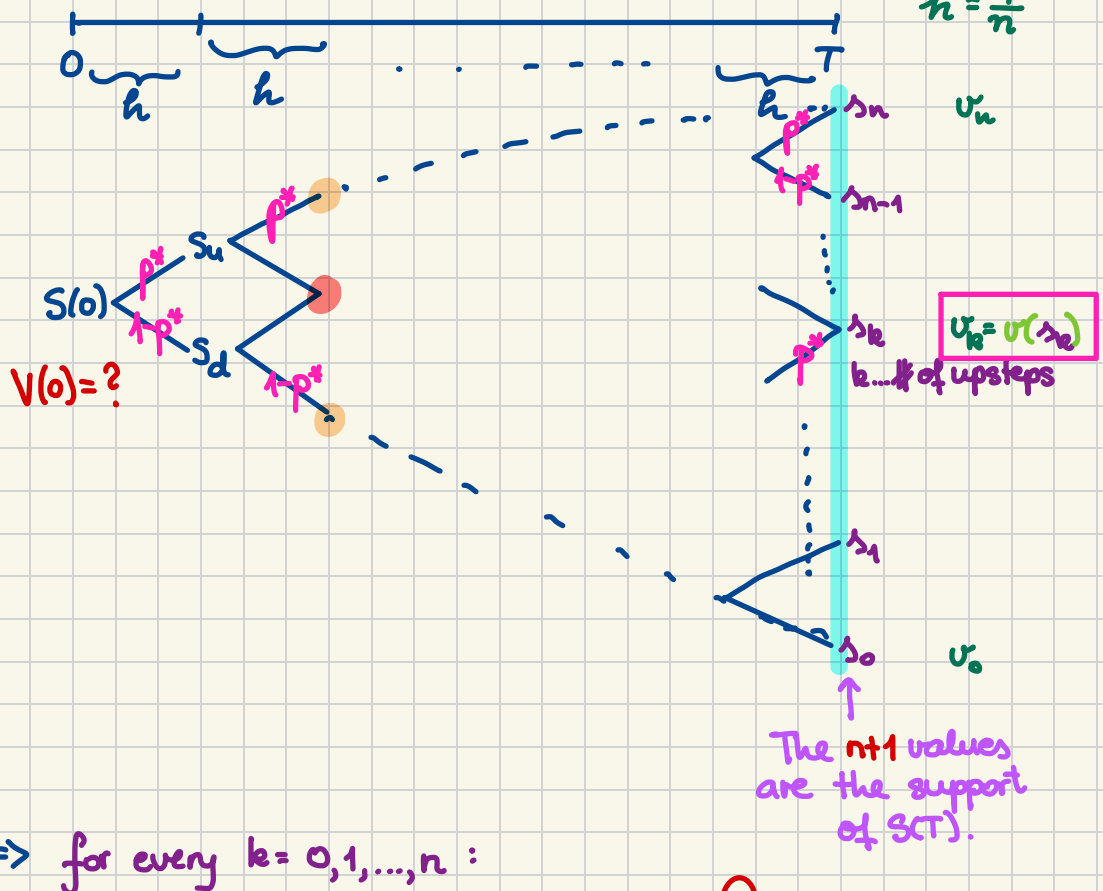
Multiple Binomial Periods.

T... exercise date of a European option } the length of each period is $h = \frac{T}{n}$

n... # of periods



$\Rightarrow$ for every $k = 0, 1, \dots, n$:

$$s_k = S(0) \cdot u^k \cdot d^{n-k} = S(0) \left(\frac{u}{d}\right)^k d^n$$

Consider a European option w/ payoff f'tion $v(\cdot)$
Then, the possible values of the payoff are

$$v_k = v(s_k) \quad \text{for all } k = 0, 1, \dots, n$$

Risk-Neutral Pricing.

$$V(0) = e^{-rT}\mathbb{E}^*[V(T)]$$

w/ $p^* = \frac{e^{rh} - d}{u - d}$

$\Rightarrow$ The risk-neutral probability of reaching the payoff v_k is

$$\boxed{\binom{n}{k}(p^*)^k(1-p^*)^{n-k}} \quad \text{for all } k = 0, 1, \ldots, n$$

$\Rightarrow$ The risk-neutral option price:

$$\boxed{V(0) = e^{-rT}\sum_{k=0}^{n}\left(\binom{n}{k}(p^*)^k(1-p^*)^{n-k} \cdot v_k\right)}$$

The University of Texas at Austin
Homework Assignment 15

M339D Introduction to Financial Mathematics

October 26, 2026

Instructions: Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Multiple binomial periods.

Problem 15.1. Consider a non-dividend-paying stock whose current price is \$90 per share. The stock has the volatility equal to 0.20. $\sigma = 0.2$

Let the continuously compounded, risk-free interest rate be equal to 0.03. $r = 0.03$

You model the evolution of this stock over the next year with a three-period **forward** binomial tree. $h = \frac{1}{3}$

What is the price of a \$95-strike, one-year put on the above stock consistent with this model?

Special Case: Forward Binomial Tree.

σ... volatility

$u := e^{rh+\sigma\sqrt{h}}$

$d := e^{rh-\sigma\sqrt{h}}$

The Risk-Neutral Probability:

$$p^* = \frac{e^{rh}-d}{u-d} = \frac{e^{rh}-e^{rh-\sigma\sqrt{h}}}{e^{rh+\sigma\sqrt{h}}-e^{rh-\sigma\sqrt{h}}}$$

$$p^* = \frac{1-e^{-\sigma\sqrt{h}}}{e^{\sigma\sqrt{h}}-e^{-\sigma\sqrt{h}}} \cdot \frac{e^{\sigma\sqrt{h}}}{e^{\sigma\sqrt{h}}} = \frac{e^{\sigma\sqrt{h}}-1}{e^{2\sigma\sqrt{h}}-1}$$

$$e^{2\sigma\sqrt{h}}-1 = (e^{\sigma\sqrt{h}}-1)(e^{\sigma\sqrt{h}}+1)$$

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} \xrightarrow{h \to 0} \frac{1}{2}$$

The shortcut ONLY for the FORWARD BINOMIAL TREE.

→: $p^* = \frac{1}{1+e^{0.2\sqrt{1/3}}} = \underline{0.4712}$

$u = e^{rh+\sigma\sqrt{h}} = e^{0.03(1/3)+0.2\sqrt{1/3}} = \underline{1.1337}$

$d = e^{rh-\sigma\sqrt{h}} = e^{0.03(1/3)-0.2\sqrt{1/3}} = \underline{0.8999}$

K=95

$S_{ddd} = S(0)\cdot d^3 = 65.59$ $\quad V_{ddd} = (95 - S_{ddd})_+ = 29.41$

$S_{udd} = S(0)\cdot u\cdot d^2 = \underline{82.63}$ $\quad V_{udd} = (95 - S_{udd})_+ = 12.37$

$S_{uud} = S(0)\cdot u^2\cdot d = \underline{104.09} > 95$ $\quad$ 0·0·money

$$V(0) = e^{-0.03\cdot 1}\left((1-p^*)^3\cdot V_{ddd} + 3\cdot p^*(1-p^*)^2 V_{udd}\right)$$

$= \dots = \underline{8.97}$ □

CCRFIR

Problem 15.2. Let the be 0.10. Let the initial price of a non-dividend-paying stock be \$100 per share. You use a five-period binomial tree to model the stock price over the next year. Let $u = 1.04$ and $d = 0.96$.

What is the price of a one-year, at-the-money European call option on the above stock?

$$\rightarrow: \quad p^* = \frac{e^{0.10\cdot(1/5)} - 0.96}{1.04 - 0.96} = 0.7525$$

K=100

$S_{u^5} = S(0)\cdot u^5 = 121.67$ $\qquad v_{u^5} = 21.67$

$S_{u^4d} = S(0)\cdot u^4\cdot d = 112.31$ $\qquad v_{u^4d} = 12.31$

$S_{u^3d^2} = S(0)\cdot u^3\cdot d^2 = 103.67$ $\qquad v_{u^3d^2} = 3.67$

$$V_C(0) = e^{-0.10}\left[21.67\,(p^*)^5 + 12.31\cdot 5\cdot (p^*)^4(1-p^*) + 3.67\binom{5}{2}(p^*)^3(1-p^*)^2\right] = ____$$

$\binom{5}{2} = \frac{5\cdot 4}{2} = 10$