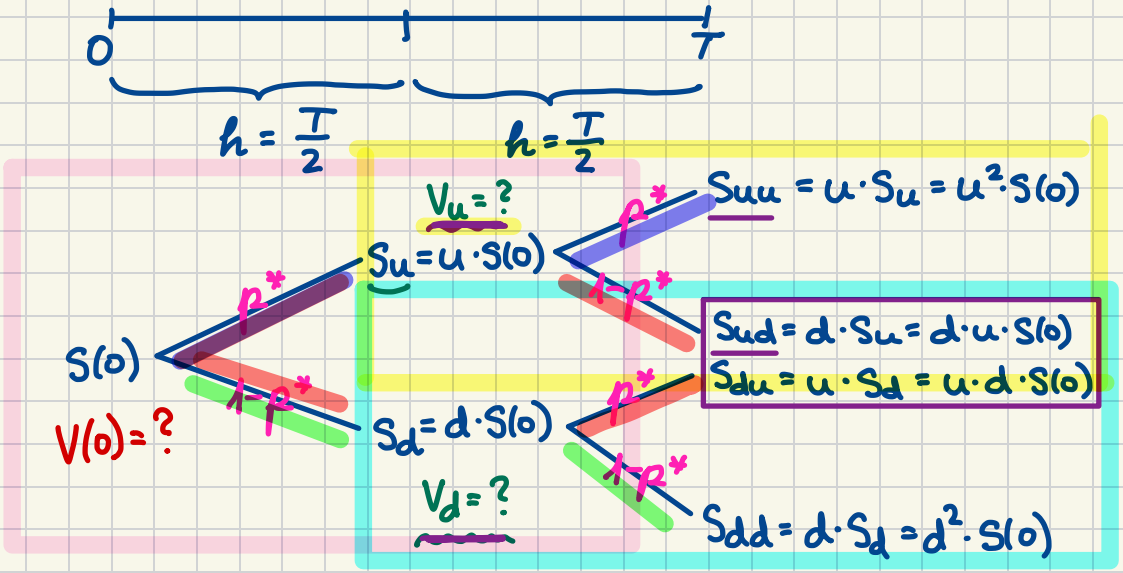
Two Periods.

$n = 2$



PAYOFF

$V_{uu} = v(S_{uu})$

$V_{ud} = v(S_{ud})$

$V_{dd} = v(S_{dd})$

populating the tree →

← pricing the option

- **up node:** replicating portfolio for the option

$$\begin{cases} \Delta_u = \dfrac{V_{uu} - V_{ud}}{S_{uu} - S_{ud}} \\ B_u = e^{-rh} \cdot \dfrac{u \cdot V_{ud} - d \cdot V_{uu}}{u - d} \end{cases}$$

$\Rightarrow$ the option's value at the up node:

$$V_u = \Delta_u \cdot S_u + B_u = e^{-rh} \cdot \left[p^* \cdot V_{uu} + (1-p^*) \cdot V_{ud} \right]$$

w/ $\boxed{p^* = \dfrac{e^{rh} - d}{u - d}}$

- **down node:** Δ_d, B_d

$$\Rightarrow V_d = \Delta_d \cdot S_d + B_d = e^{-rh} \cdot \left[p^* \cdot V_{ud} + (1-p^*) \cdot V_{dd} \right]$$

- **ROOT NODE:** $\Delta_0 = \dfrac{V_u - V_d}{S_u - S_d}$

$$B_0 = e^{-rh} \cdot \frac{u \cdot V_d - d \cdot V_u}{u - d}$$

$$\Rightarrow \boxed{V(0) = \Delta_0 \cdot S(0) + B_0}$$

From the "risk-neutral perspective":

$$V(0) = e^{-rh} \cdot [p^* \cdot V_u + (1-p^*) \cdot V_d]$$

$$= e^{-rh} \cdot [p^* \cdot e^{-rh} \cdot (p^* \cdot V_{uu} + (1-p^*) \cdot V_{ud}) + (1-p^*) \cdot e^{-rh} \cdot (p^* \cdot V_{ud} + (1-p^*) \cdot V_{dd})]$$

$$= (e^{-rh})^{2h=T} [(p^*)^2 \cdot V_{uu} + 2 \cdot p^*(1-p^*) \cdot V_{ud} + (1-p^*)^2 \cdot V_{dd}]$$

Risk-Neutral Expectation of the Payoff

Generally: $\boxed{V(0) = e^{-rT} \mathbb{E}^*[V(T)]}$